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RAILROAD ENGINEERING

PART I

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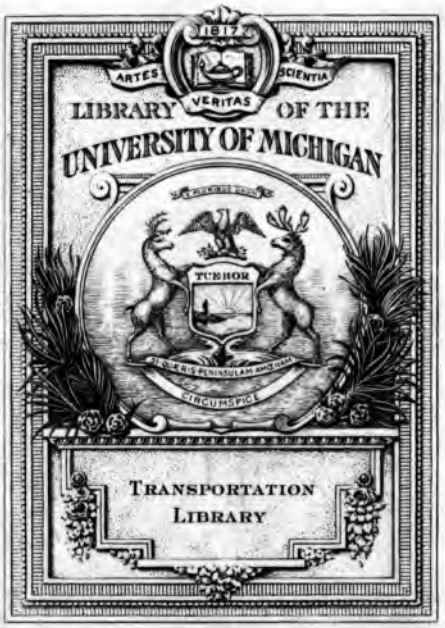
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RAILROAD ENGINEERING

PART I

INSTRUCTION PAPER

PREPARED BY

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AMERICAN SCHOOL OF CORRESPONDENCE

CHICAGO ILLINOIS

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RAILROAD ENGINEERING.

PART I.

RAILROAD SURVEYS.

1. General Principles. The engineer should have first, a thorough appreciation of the objects to be accomplished by the surveys. He should realize that, except in the rare cases where it is difficult to find *any* practicable line, very little engineering training or ability is required to lay out a line over which it would be physically possible to run trains. A line as laid out may violate all rules of location, may be expensive to operate and have disadvantages which will discourage traffic, and yet trains *can* be run over it. From the infinite number of possible locations, the engineer must select the location which best satisfies the various conflicting interests. His value as an engineer depends on his ability to interpret the natural conditions and design the line accordingly. This ability is only obtained by a thorough knowledge of the whole subject of railroad engineering, supplemented by practical experience. It is therefore true that many of the following statements will not be thoroughly appreciated until the student has covered the whole subject and then reviews it.

2. Conflicting Interests. There are several classes of interests, which are generally more or less conflicting, which affect the location of every line.

(*a*) *The initial cost should be a minimum*, but the cheapest road generally has sharp curvature, steep grades and inconvenient location.

(*b*) *The operating expenses per train mile should be a minimum*, which is generally equivalent to saying that the curvature should be light and the grades low, but this is usually unobtainable except at great cost.

(*c*) *The location should be convenient to sources of traffic* so that the maximum traffic will be obtained, but this is generally very costly.

A little study will show the frequent conflict of the above conditions. When a proposed location evidently combines the above interests advantageously, instead of bringing them into conflict, then there is no doubt as to the proper location, unless it affects unduly the adjacent location. The best engineering ability is a cheap investment when deciding on a location which requires a delicate balancing of the claims of several possible routes, each with its own combination of greater or less initial cost, greater or less operating expenses, and greater or less effect on the probable revenue of the road.

RECONNOISSANCE SURVEYS.

3. Essential Problem. From the above considerations it may readily be seen that the first survey to be made (called the reconnoissance survey) consists essentially of a broad examination of the country through which the road is expected to pass. Business considerations usually predetermine that the road is to connect certain termini and also pass through certain intermediate important towns or cities, but the problem consists in finding the best route between the predetermined points. When two consecutive predetermined points lie in the same valley or on the same bank of a river too large to be easily bridged, the location is self-evident. If the river is smaller, easily bridged, has sharp bends, with variable banks and important towns on either bank, it will usually require a close examination of each bank to determine where to cross if at all. When the two points are many miles apart, lie in different valleys, and are therefore separated by one or more summits, the selection of the best route becomes more and more complicated as the number of possible routes becomes greater. It is generally true, although not invariably, that a cross-country route which includes the lowest summits and the highest *low points* (such as river crossings) will give the best grades. Since the "ruling grade" is the most important physical consideration for the engineer, as will be developed later, the chief work of the reconnoissance survey (apart from considerations of probable traffic) is the determination of the elevations of summits and sags and the distance between them, together with the constructive character of the country.

4. Utilization of Existing Maps. The U. S. Geological Survey has already published contour maps of a large part of the country which enable an engineer to select a line with even greater ease and certainty than he can from a reconnoissance map made for the purpose (as usually made), since the U. S. G. S. maps show the *whole* country and enable the engineer to rapidly compare a dozen suggested routes instead of confining his attention to the (usually) limited area of the special map. The errors of the U. S. G. S. maps will seldom if ever be sufficient to vitiate the accuracy of the preliminary route laid out from them. Usually a brief study of the map will demonstrate that one (or perhaps two or three) general route has advantages so pronounced over all other possible routes that the choice is immediately made or is at least reduced to the comparison of two or three lines which are so nearly equal that closer and more detailed surveys are necessary to decide between them. County atlases are usually sufficiently accurate for reconnoissance purposes to the extent of giving the relative horizontal positions of governing points of the survey. Elevations may be determined (as described later) and plotted on these maps.

5. Surveying Methods. When reliable contour maps are unavailable, some of the following methods may be used to fill out existing maps or to make a complete reconnoissance survey. The essential point is the rapid determination of those details from which one route is shown to be superior to another. Nothing useless should be surveyed and no time should be wasted on an unnecessary degree of accuracy. The physical characteristics of two routes have usually such differences that they are apparent even with rapid and approximate methods of surveying. If two routes are so nearly equal that a decisive choice cannot be made from the results of reconnoissance surveys, it shows that a more accurate survey should be made of both routes.

6. Elements. The three elements of the survey of any line are (*a*) the length, (*b*) the direction, and (*c*) the slope or the relative elevation of the two ends. *Distance.* The length is sometimes determined with sufficient accuracy by pacing, the steps being counted with a pedometer. In an open prairie country, where a buggy may be run, an odometer attached to a wheel will count the

revolutions. An odometer on a wheel, attached to a frame and trundled like a wheelbarrow, has been used for the same purpose. A large telescope, mounted with a universal joint on a very light tripod, and fitted with stadia wires so adjusted that distances of 2,000 or even 2,500 feet can be read to the nearest 10 feet on a 10-foot rod, will give the distances between widely separated stations with sufficient accuracy and extreme rapidity. *Direction* may be obtained with sufficient accuracy with a compass—even of the pocket type. *Leveling.* Spirit leveling is too slow and expensive for the rapid surveying here required. If stadia methods are used with an instrument provided for reading vertical angles, the inclination of all lines may be observed and the elevations of all stations computed. A still more rapid method of observing differences of elevation with sufficient accuracy for the purpose is found in the use of an aneroid barometer, supplemented by another aneroid or preferably by a mercurial barometer. The mercurial, or the office aneroid, is kept at some office whose elevation is known and observations are regularly taken (say every half hour) during the period when observations are being taken in the field with the field aneroid. The field aneroid is taken to each place, within a range of several miles, where elevations are desired. At each point there should be noted (see the form of notes below) the time, the described location, the aneroid reading and the temperature. If possible, duplicate readings should be taken on the trip to and from the office on all important points. The elevations of succeeding office locations made, may be determined with the field aneroid if necessary, but of course extra care should be taken with such work.

Aneroids are usually "compensated for temperature," *i.e.*, so adjusted that they will give a true reading regardless of temperature. If an aneroid has not been so adjusted, it should be carefully compared with a standard mercurial barometer under widely varying conditions of temperature and a tabular form should be made out for that aneroid showing the correction to be applied at any given temperature. On account of the expansion of mercury with temperature, and also the expansion (at a different rate) of the tube and cistern, all readings of the mercurial barometer must be "reduced to 32° F.," *i.e.*, reduced to the reading it would have, if the temperature of the instrument were 32° F. This is readily ac-

complished by means of Table XI.* At the office, each half-hourly observation should include the time, the reading of the scale showing the height of the mercury, the reading of the "attached thermometer" (the thermometer attached to the mercurial) and also the temperature of the external air. When the mercurial is indoors these two temperatures may differ somewhat. When reducing the observations interpolation should be made if necessary between the reduced office observations to determine the probable reading of the mercurial at the time of any given field observation. Determine from Table XII* the heights corresponding to the field reading and reduced office reading for each pair of observations. Their difference is the *approximate* difference of elevation of the office and of the place of the field observation. If necessary this may be corrected by an amount equal to the approximate difference of elevation times a coefficient derived from Table XIII.* This coefficient is found opposite the number which gives the *sum* of the temperatures in the field and outside the office. The correction is frequently too small to be noticed. An approximate calculation will often show this, or will give a solution to the nearest foot, which is amply accurate. An aneroid, no matter how perfect, will seldom agree exactly with a mercurial barometer, and even if adjusted to the same reading will soon indicate some discrepancy. It is therefore better to leave the adjustment undisturbed and apply corrections. The aneroid should therefore be compared with the mercurial before leaving headquarters for a day's work, and the readings of both and their *difference* should be recorded. Immediately after returning from the day's work the aneroid should again be compared. The absolute reading of the mercurial will probably be higher or lower, but the *difference* should be nearly the same, although it is found that an aneroid will lag somewhat behind its true reading, especially if it has been subjected to an extreme variation of pressure. All the field readings of the aneroid should therefore be corrected by the *mean* of the initial and final differences. The method and the above explanation may be illustrated by the following numerical examples:

7. Examples. 1. Given a reading of 28.692 on a mer-

*See Webb's "Trigonometric Tables," published by American School of Correspondence, Chicago, Ill. Price, 50c.

curial barometer, what is its reading when reduced to 32° F., the reading of the attached thermometer being 68.5° F.? In Table XI*, under 28.5 and opposite 68°, we find – .101. Under 29.0 and for 68° we find – .103. For 28.692 and 68° it evidently should be – .102 (to the nearest thousandth). Similarly for 28.692 and 69° we may derive – .105. For 28.692 and 68.5° it would be the mean or – .1035, which we will call – .103 since it is useless to compute the correction closer than the nearest thousandth. Then since the correction is – .103, the corrected reading should be 28.589. With a little practice the interpolations, when necessary, may be made in far less time than it takes to describe it.

2. Verify the following reductions:

Bar. reading.	Temp.	Reduced reading.
26.426	58° F.	26.356
27.892	78.5	27.767
28.475	85.	28.330
30.847	48.5	30.792

3. Reduce the following readings: 27.294, 47°; 29.462, 87°; 26.230, 78.5°; 25.241, 62°; 26.481, 75°; 29.625, 89.5°; 30.942, 88.5°; 29.784, 46.5°; 28.386, 48°; 27.942, 74.5°.

4. Compute the barometric elevation corresponding to a reading of 28.589. From Table XII* the reading for 28.5 is 1397 and the difference for .01 is – 9.5; therefore, for .089 the correction will be $-9.5 \times 8.9 = -84.55$, or in whole numbers – 85. Then 1397 – 85 = 1312, the corrected reading.

5. Verify the following elevations from the reduced readings: 26.356, 27.767, 28.330, and 30.792; *i.e.*, 3528, 2107, 1560, and – 710.

6. Compute the barometric elevations corresponding to the *reduced readings* found by solving Example 3.

7. With an approximate difference of elevation of – 136 feet and field and office temperatures of 62° and 67°, what is the true difference of elevation? 62 + 67 = 129. For 129° the coefficient is (by interpolation) + .0357. $136 \times (+ .0357) = + 4.8552$. For this slight difference of elevation, the coefficient is *far* more accurate than necessary, and of course the correction is called + 5.

*See Webb's "Trigonometric Tables," published by American School of Correspondence, Chicago, Ill. Price, 50c.

The difference of elevation should be *increased* by 5, but the difference is essentially negative. Therefore we have as the correction $-(+5)$. The true difference of elevation is $-136-(+5)=-141$.

8. The following example shows not only the method of recording the observations but also the complete solution of a problem.

Time.	Mercurial barometer.	Attached thermometer.	Reduction to 32° F.	Corrected reading.	External thermometer.
7:00 A. M.	28.692	62°	−.087	28.605	60° F.
:30	.724	64	−.092	.632	62
8:00	.756	66.5	−.099	.657	64
:30	.782	68	−.102	.680	65
9:00	.824	69	−.105	.719	66

The observations taken in the field at this time were as given in the first four columns of the following tabular form. The other columns are computed later in the office.

(Left-hand page of notes.)

Time.	Place.	Aneroid.	Therm.	Corrected aneroid.	Corrected mercurial.
7:00 A. M.	Office.....	28.743	62°	.....	28.605
7:20	R. R. Junction.	.769	63	28.631	.623
8:10	Blue River	.860	65	.722	.665
8:50	Saddle in Beanpole ridge	.522	66	.384	.706

(Right-hand page of notes.)

Ext. temp. office.	Approx. field reading.	Approx. office reading.	Difference.	Correction for temp.	Difference of elevation.
....	.....	.....	.....	.....	.....
62°	1273	1280	−7	0	−7
64	1186	1240	−54	−(+2)	−56
66	1508	1201	+307	+12	+319

8. Low Ruling Grades. It will be developed later that a low ruling grade is of prime importance. The approximate value of the ruling grade is determined from the reconnoissance survey. If the country is mountainous, it may be necessary to "develop" the line in order to reduce the grade. "Development" here

Fig. 1. Georgetown Spiral.

means a deliberate increase in the length of the line between two predetermined points so that the rate of grade shall be as low as desired. The Georgetown spiral, shown in Fig. 1, is perhaps the most famous example in this country of this general method. A study of the course of the track will illustrate several methods of taking advantage of the topography and attaining a considerable elevation although the grade is kept low.

PRELIMINARY SURVEYS.

9. General Object. The reconnoissance survey has shown that the best location for the road will lie somewhere through a certain belt of country. In some places this belt may be very narrow, *i.e.*, certain topographical features will determine that the road *must* pass through a strip but little if any wider than the roadbed requirements. In other places the choice of possible location is so widened that it is necessary to survey everything within reach of the backbone line of the survey. The willingness or financial ability of the company to ignore minor topographical considerations and incur heavy expense in order to obtain economic advantages, may also widen the area of possible location. As a general statement, the width of the belt surveyed should so vary as to include all practicable locations along that general route.

10. Cross Section Method. A broken line is run which shall lie as near the expected location line as possible. The bearing and length of each segment of the broken line is determined and also all essential topographical features on either side. Bearings are sometimes taken only with a compass, which has the advantage of great rapidity but lessened accuracy. For more accurate work, true azimuth is carried along by means of back sights at previous stations. The azimuths between stations should be checked by means of needle readings. It is advisable to determine exact azimuth at the beginning of a survey and at intervals of a few miles. This may be done by observations on Polaris (see Plane Surveying, Part II, Pages 95 to 97), or still better, by solar observations which may be taken with great accuracy at any time of day. Set stakes at each even 100 feet. In general the instrument stations will not occur at the even 100-foot distances, but the odd distance should always be carried on to the next course. The

stakes should be about fifteen inches long and about one-and-one-quarter inches square. Stakes with a cross-section of one inch by one-and-one-half inches are preferred by some. The stakes indicating the 100-foot stations should be driven to within five inches of the ground. Stakes indicating the locations of the transit (called hubs) should be driven flush with the ground. A "witness stake" should then be driven three feet to the right and on this stake should be marked the station number and the "plus distance"; *e.g.*, the stake might show 137 + 46, which would indicate that the stake was 46 feet beyond Sta. 137, and 13,746 feet from the starting point. Station stakes should be marked with the station number on the *rear* side of the stake. Immediately following the transit party, the level party should obtain the elevations above the datum plane of all stations and substations, ridges, sags, river banks and any point where the profile changes abruptly.

11. Cross Sectioning. Use a Locke level, resting on a five-foot stick, a 50-foot tape and a ten-foot rod graduated to feet and tenths. The cross-section party takes cross sections (usually) at every 100-foot stake, the cross section being made perpendicular to the backbone line of the survey at that place, as is indicated by the dotted lines in Fig. 2. It is desired to plot on the map contours at each five-foot interval above the datum plane. Let Fig. 3 represent a typical cross section. Set the level (on its five-foot stick) at the stake S. The elevation of this stake given by the level party is (say) 169.4. The level therefore has an elevation of 174.4. If the level rod is moved up hill until it is found (by trial) that 4.4 mark is on a level with the telescope, then the base of the rod must have a level of 170 and must be on the 170-foot contour. Measure the distance *horizontally* from stake to rod and record as shown in Fig. 4. Leaving the level rod at that point, carry the stick and level up the hill until a level line strikes the top of the rod. The base of the stick is evidently on the 175-foot contour. Measure and record the distance as before. Carry the level rod to that point and in a similar manner determine the 180-foot contour if desired. The 165-foot contour is evidently 9.4 feet below the telescope when on the 5-foot stick at the center stake. The distance from the center to the 165-foot contour can thus be found. Lower contours can be similarly obtained. The results

should be plotted in a note-book ruled in quarter-inch squares, each side of a square representing 25 feet. The work will then be plotted on the scale of 100 feet per inch. If the successive stations are plotted *up* the page, the drawing will correspond with the points when looking ahead along the line. After plotting each section, the corresponding contours should be connected to form a sketch like Fig. 4. The crossing of the main line by a contour may be similarly determined. Fig. 4 is simply an enlarged detail of a sketch like Fig. 2. Although the Locke level is incapable of

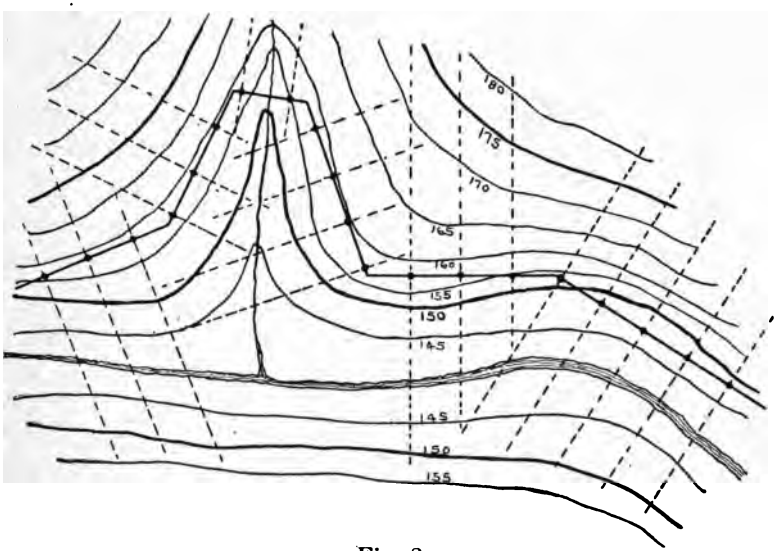


Fig. 2.

accurate leveling work, any error that may be made by the above method is confined to the station where it occurs and is not carried on and made cumulative. With reasonable care such inaccuracies can be kept within desired limits, while the rapidity is far greater than a more accurate method.

12. Stadia Method. This consists simply of a stadia survey of a long and narrow belt of country by the same general methods as those employed in ordinary stadia-topographical surveys. One advantage of this method is that the levels can be carried along very successfully as a part of the stadia work, if particular care is taken to always obtain practical agreement in the vertical angles

for the foresight and backsight between consecutive stations. This will generally permit more rapid work, as the progress of the whole party is sometimes limited by the progress of the level party. The added cost of the level party is also saved. It is here assumed that the details of stadia work have already been studied and therefore no further discussion will be given of this very simple application of the general method. As in the previous method, the primary object of the survey is the preparation of a map showing the contours and required topographical features over the desired area.

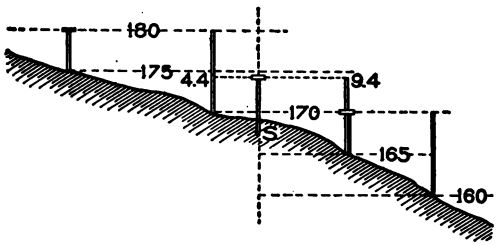


Fig. 3.

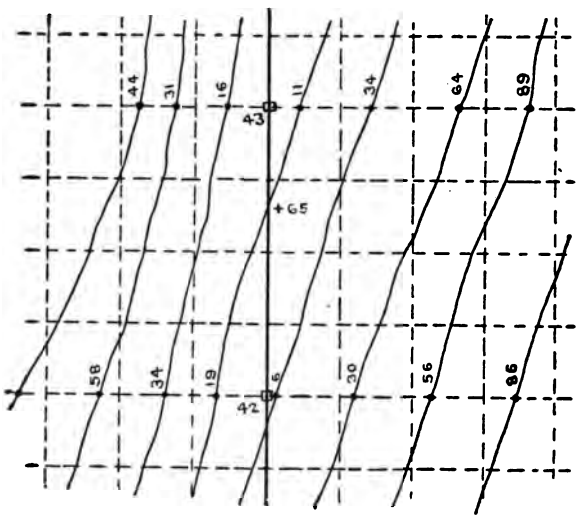


Fig. 4.

13. **Party Required.** It has been forcibly said that the only duty of the *chief-of-party* is to "keep his eyes open". The selection of the best route for a road so depends on a close study of the country that if the chief-of-party is required to do the work

of transitman, as is sometimes the case, the work of either position is apt to suffer. The work of the transitman is so exacting that he should not be required to spend any time in studying out a route. Beside these two, there should be two flagmen, two chainmen, one stakeman and two or more axemen—depending on the wooded character of the country. On stadia surveys the flagmen and chainmen may be replaced by two or more rodmen; it is also economical to have a recorder, as it facilitates the progress of the whole party. The cross-section party should consist of a levelman, recorder, and two tapemen. This party *can* be cut down to three, or in an emergency two, but it is uneconomical in the long run. The level party will consist of a leveler and rodman. If the party is camping out, a cook and one or more teamsters will usually be required to handle the camp equipage, as it is unwise to require the surveyors to spend their time in such work.

14. Re-surveys. Much of the defective location of railroads is due to (1) deciding hastily on a general route, (2) then surveying a line through the belt with great detail and accuracy, (3) then locating the line substantially as first surveyed, because the line is fairly good (or at least not very bad), and also because of an unwillingness to throw away the detailed work of a large party for several weeks. Frequently a great amount of unnecessary and wholly useless detail is surveyed and plotted during the reconnoissance and preliminary surveys. These surveys should only include those salient facts which instantly stamp a route as being inferior or superior to another. Usually the general location of a large part of a route is self evident or may be determined after a brief examination. But there are generally places along the line where for a few miles a hasty examination of two or three lines is not only justifiable but is the only proper course. Two or more of these short loops may show advantages so evenly divided that a more elaborate survey is necessary to decide between them. Even after the location survey has been made, or even after construction has begun, changes are often proper, but if the preliminary surveying has been well done only minor changes should be needed. A few hundred dollars spent on extra surveying is a wise investment considering the great probability of an immediate saving of as many thousands in construction or of an

operating advantage whose annual value might be as great as the cost of the extra surveying.

LOCATION SURVEYS.

15. Selecting a Route. Much of the railroad location of the country has been done by picking out the line on the ground, even making it follow in places the backbone line of the preliminary survey, running from one course to the next by means of suitable curves. In the hands of a good engineer the method is not necessarily very bad, but it is much improved by the following modification. *Paper location.* The work of the preliminary survey is carefully plotted from the transit notes and cross-section book to a scale of 200 feet per inch. On this map may be plotted one or more trial location lines. Each of these consists of circular curves joined by tangents. The location line must pass through any predetermined points and yet join them by lines which will give the best location, considering the conflicting interests as described in section 1. Within the limits of the preliminary map several locations are generally possible and one great element of the value of such a map lies in the ease with which several routes may be laid out and compared. Profiles may be drawn for each line laid down by noting the intersection of the line with each contour. Drawing on the profile the required grade line will give a relative idea of the amount of earthwork required. The method is especially valuable when "development" is necessary. Although such a line must sometimes be laid out by a bold and apparently unsystematic trial of a route, yet some approach to a systematic solution may be made as follows: Assume that the maximum ruling grade has been determined as 1.2 per cent, and that the contours have, as usual, a five-foot interval. It will require 417 feet of 1.2 per cent grade to rise five feet. Set a pair of dividers so that they will step off spaces of 417 feet on the map. Starting on a contour at the required beginning of a grade, swing the dividers so that they will just reach the next contour and continue to step off such spaces. Joining these points, such a line would be a purely surface line, would probably be very crooked and otherwise unsuitable, but it probably would be suggestive of a practicable route. After locating on the map the best

obtainable line, it should then be transferred to the ground. Measure to scale the lengths of all "tangents" (the straight lines joining the curves), and the radii and lengths of all curves. Instead of scaling off the *length* of a curve, it may be more accurate to measure with a protractor, or with a scale of chords, the angle between the tangents at each end of the curve, and from the angle and the radius compute the length. Usually the located line will lie fairly close to the preliminary line—close enough so that tie lines may readily be run between them. These should be scaled from the map. To prevent the accumulation of error due to inaccuracies, the length (or radii) of curves or the length of tangents should be altered if necessary so as to make the location check on the ground with the positions of the stakes of the preliminary survey. The method of making such modifications will be taken up later.

16. Surveying Methods. Only the most precise work with a transit can be tolerated. The compass needle is only to be used as a check, but its use for this purpose should be insisted on, as it frequently detects a gross error. Transit stations should be marked by "hubs" and "witness stakes" (Section 10). Reference stakes should also be set at places as near as possible to the principal stations and yet outside of the line of all earthwork operations, so that at any stage of the construction the positions of the original stakes may be easily recovered. The link chain as a measurer has now been practically discarded for the steel tape. Fractions of a foot are measured in tenths and hundredths rather than in inches. The *personnel* of the party will be almost identical with that of the preliminary survey party except that the cross section party will be replaced by the slope-stake party, whose duties are similar, but who generally use a level on a tripod rather than a hand level. The description of the duties of the slope-stake party will be deferred to a later chapter. The leveling party should establish "bench-marks" at frequent intervals along the line. A spike driven in the roots of a large tree is one of the best and easiest established of marks in rural districts. A mark on any large masonry structure, such as a bridge abutment or a building, should be obtained when possible. Levels should be taken to hundredths of a foot on turning points and bench marks. Some engineers

read to thousandths of a foot, but when it is considered that one division of a level bubble usually corresponds to 30″ of arc, and that at a distance of 150 feet a movement of 30″ of arc will correspond to .0218 foot on the rod, an error of level amounting to a very small fraction of a division will make an error of several thousandths or even a hundredth. Therefore unless unusual care is taken in handling the level, it is a useless refinement to read the rod to thousandths. In reading elevations of the surface of the ground, the nearest tenth of a foot is sufficiently accurate. The complete details of location surveys can only be appreciated after the subject of railroad curves has been studied, and they will not therefore be further elaborated here.

SIMPLE CURVES.

17. Method of Measurement. The alignment of a track is the geometrical form of the line midway between the two rails. Such a center line may be a straight line, a simple curve or a curve of double curvature, but it simplifies matters to consider always the *horizontal projection* of such lines. Their vertical projections are considered separately when it is necessary. Curves are sometimes designated by their radius or by the degrees and minutes subtended by a unit chord. Nearly all railroad curves have such long radii that it is impracticable to use the center. Therefore all work is done at the circumference in accordance with geometrical principles which will now be described.

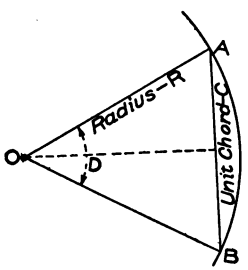


Fig. 5.

If AB, Fig. 5, is a chord of unit length, then D is called the *degree of curve* for the radius R.

$$\text{AO} \sin \frac{1}{2} \text{D} = \frac{1}{2} \text{AB} = \frac{1}{2} \text{C}.$$

$$\therefore \text{R} = \frac{\frac{1}{2} \text{C}}{\sin \frac{1}{2} \text{D}} \qquad (1)$$

which becomes by inversion

$$\sin \frac{1}{2} D = \frac{C}{2R} \qquad (2)$$

The length of the unit sub-chord varies somewhat with custom. The almost invariable practice in the United States is to use a unit chord length of 100 feet. Substituting C = 100 in equation 1, and successively assuming values from 0° 01′ up to 12° 0′ varying by single minutes, and with larger intervals for higher degrees which are very seldom used, the radius of almost any curve may be tabulated for ready and convenient use. Such a table is found in Table I*, which also gives the logarithm of each radius. A very common rule, which is approximate but accurate enough for many uses, is as follows, using the same notation as before:

$$R = \frac{5730}{D} \qquad (3)$$

18. Sub-Chords. It often becomes necessary to lay off a chord length which is less than 100 feet and to know the angle subtended at the center. Since a chord is shorter than its arc, it also follows that the sum of the four equal chords in Fig. 6 is also shorter than the total arc although they are evidently longer than the 100′ chord. But it is found more convenient to say that the chord has a *nominal* length (in this case) of 25 feet. As in equation (2) we may derive

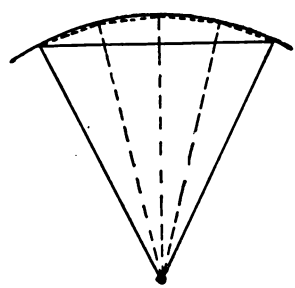


Fig. 6.

$$\sin \frac{1}{2} d = \frac{c}{2R} \qquad (4)$$

In which d is the angle subtending the sub-chord whose *true* length is c. By inversion we have

$$c = 2 R \sin \frac{1}{2} d \qquad (5)$$

Calling the *nominal* length c', we have the proportion

$$c' : 100 :: d : D$$

*See Webb's "Trigonometric Tables," published by American School of Correspondence, Chicago, Ill. Price, 50c.

EXAMPLES FOR PRACTICE.

1. What is the true length of a chord of a 3° 30′ curve whose nominal length is 40 feet? From the above proportion, $d = \frac{40}{100} D = 0.40 \times 3.5 = 1.4° = 1° 24'$. Substituting in equation 5, we have

$$c = 2 \times 1637.3 \times \sin \frac{1}{2}(1° 24') = 40.005.$$

Note that the *excess* over 40 feet is very small—about one-sixteenth of an inch. It is always small for low degrees of curvature. In the following example it is far greater.

2. What is the true length of a chord of a 12° curve whose nominal length is 60 feet? *Ans.* 60.070. In this case it would be a gross error to neglect to allow for this difference.

19. Length of a Curve. The length of a curve is always considered to be the quotient of $100\Delta \div D$, in which Δ is the total central angle of the curve or the angle between the terminal tangents. The mean length of the two rails of a curve is always a little in excess of this, but the excess is always so small that it has no practical importance. It merely adds an insignificant amount to the length of rail required. *Example.* A 4° curve begins at Sta. 16 + 80 and runs to Sta. 21 + 35. The *nominal* length of the curve is 455 feet. The actual arc (which is the mean of the two rail lengths) is

$$4.55 \times 4° \times R \times \frac{\pi}{180°} = 455.09$$

which shows that the excess in this case = .09 foot, a little over an inch.

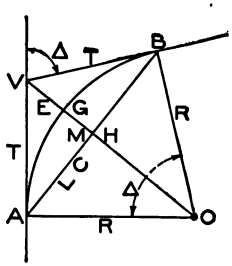


Fig. 7.

20. Elements of a Curve. The following fundamental relations apply to all curves. See Fig. 7. The beginning of the curve, A, is called the *point of curve*, PC. The other end of the curve at B is called the point of *tangency*, PT. The intersection of the two tangents is called the *vertex* (V). The *central angle*, Δ, is the angle at V between the tangents, and it is equal to the angle at the center, O, between the radii drawn to the PC and PT. The

two equal tangents AV and BV are called *tangent distances*, T. The chord AB is called the *long chord*, LC. The distance HG from the middle of the long chord to the middle of the arc is called the middle ordinate, M. The distance GV from the middle of the arc to the vertex is called the *external distance*, E. From trigonometry the following commonly used relations are easily derived.

$$T = R \tan \frac{1}{2}\Delta \qquad (6)$$

$$LC = 2R \sin \frac{1}{2}\Delta \qquad (7)$$

$$M = R \operatorname{vers} \frac{1}{2}\Delta \qquad (8)$$

$$E = R \operatorname{exsec} \frac{1}{2}\Delta \qquad (9)$$

(*Note.* The *versed sine*, abbreviated to *vers*, and the *external secant*, abbreviated to *exsec*, are trigonometrical functions which are not commonly used except in railroad work, and some works on trigonometry omit their discussion. An inspection of the figure readily shows that vers $a = 1 - \cos a$, and that exsec $a = \sec a - 1$.)

From trigonometry we may derive the general equation that $\tan a \div \operatorname{exsec} a = \cot \frac{1}{2} a$. Therefore, by dividing equation 6 by equation 9 and transposing we obtain

$$T = E \cot \frac{1}{4} \Delta \qquad (10)$$

21. Elements of a 1° Curve. The various elements of a curve are exactly proportional to the radius and nearly proportional to the degree of curve. Therefore if the tangents, external distances and long chords are computed from equations 6, 7 and 9 for various values of Δ from 1° to 91°, varying by 10′, then an approximate value for any degree of curve and value of Δ may be found by taking out its value for a 1° curve (by interpolation if necessary) and then dividing that value by the degree of curve. For low degrees of curvature the inaccuracy of this method is usually small

enough to be neglected. Even for sharper curvature the values obtained are accurate enough for approximate work. For absolutely accurate values equations 6 to 9 should be used, but the tabular values, found in Table II*, may always be used as a check.

22. Numerical Examples. 1. What is the tangent distance of a 3° 10′ curve whose central angle is 16° 26′?

Solution.

$$\log R = 3.25757$$

$$\frac{1}{2}\Delta = 8° 13', \log \tan = 9.15956$$

$$\text{Tangent} = 261.30 \quad \log 261.30 = 2.41714$$

Approximate solution. Interpolating in Table II* between the values for $\Delta = 16° 20'$ and 16° 30′ we have the value 827.36 as the tangent distance for a 1° curve when the central angle is 16° 26′. Dividing 827.36 by 3.1666 (3° 10′) we have 261.27 as the approximate value. The inaccuracy is about one-hundredth of one per cent or in absolute value about three-eighths of an inch.

2. Compute the external distance and the long chord for the above curve, both accurately and approximately.

3. Two tangents make an angle of 18° 24′. It is desired to run a line which shall pass 21.2 feet from the vertex of the curve. What is the required radius and the resulting tangent distance? *Indicated solution.* The known quantities are E and Δ; from equation 10 we may derive T; then with T known and Δ a given quantity, we may compute R by an inversion of equation 6.

METHODS OF FIELD WORK.

23. Location of Points by Deflections. The angle between a tangent to a curve at any point and a secant from that point to any other point of the curve, is measured by one-half of the arc between those points. It is also equal to one-half of the angle between the radii to those points. On this fundamental geometrical proposition depends the whole science of circular-curve location. As a corollary, the angle between two secants intersecting on a point of the curve is measured by one-half of the intercepted arc or by one-half of the angle between the radii drawn to the ends of the intercepted arc. Applying these statements to Fig. 8 we have

*See Webb's "Trigonometric Tables," published by American School of Correspondence, Chicago, Ill. Price, 50c.

$$\text{TO}a = \frac{1}{2}\,\text{OC}a$$

$$a\text{O}b = \frac{1}{2}\,a\text{C}b$$

$$b\text{O}d = \frac{1}{2}\,b\text{C}d$$

If Oa = 100 feet, then by definition, the angle OCa = D, and the angle TOa = $\frac{1}{2}$D. Likewise if the chord ab = 100 feet, then the angle aCb = D and the angle aOb = $\frac{1}{2}$D. bd is a subchord subtending the angle d, and the angle bOd = $\frac{1}{2}$ d. Therefore if a transit is set up at the point O, any point of the curve may be determined by measuring the proper chord length from O in a direction determined by swinging an angle from the tangent OT equal to *one-half* of the angle measured at C between O and the desired point. But the measurement need not be made directly from O if other points have already been determined; b may be determined from a and d from b. Since it is generally impracticable to locate more than 500 feet of curve from any one point, on account of natural obstructions (and sometimes the distance is very short), the transit must be moved up to a new station already established on the curve. But the same principles will apply and may be repeated indefinitely.

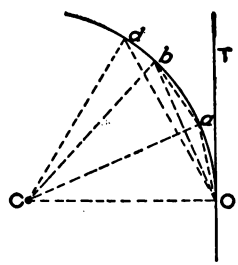


Fig. 8.

24. Computing the Deflections. If the point of curve is less than 100 feet from the last regular station, the remainder of the 100 feet must be laid off as a subchord. One-hundred-foot chords are set off until a station is reached which is within 100 feet of the end of the curve or (numerically) until the degrees of central angle remaining is less than D. That remainder is the angle for the final subchord. The foregoing may be illustrated by a numerical case: A 4° curve is to begin at Sta. 24 + 40. The central angle is 18° 40′. Compute the deflections. The first sta-

tion point is 60 feet beyond the point of curve. The subchord angle is therefore $\frac{60}{100} \times 4° = 2.4° = 2° 24'$. The deflection from the tangent is one-half of this or 1° 12′. The deflection for the P.T. is one-half of the total central angle or 9° 20′. Subtracting 1° 12′ we have left 8° 08′, which will allow for four deflections of 2° each and 0° 08′ over, which will require a chord $= \frac{0° 08'}{2°} \times 100$ $= 6.67$ feet. The curve will therefore end at Sta. 29 + 6.67. This may be verified or otherwise computed as follows: $\frac{18° 40'}{4°}$ $= 4.66667$, the total *nominal* length of the curve in station lengths of 100 feet. That is, the length will be 466.67. The first subchord is 60 feet; then four chords of 100 feet; then a final subchord of 6.67 feet. The deflections may be tabulated as follows:

P.C. Sta. 24 + 40			0°
25	0°	+ 1° 12′	= 1° 12′
26	1° 12′	+ 2°	= 3° 12′
27	3° 12′	+ 2°	= 5° 12′
28	5° 12′	+ 2°	= 7° 12′
29	7° 12′	+ 2°	= 9° 12′
29 + 6.67,	9° 12′	+ 0° 08′	= 9° 20′, which is one-half of 18° 40′ as it should be.

25. Instrumental Work. The above numerical case is comparatively simple. When the degree of curve is an odd quantity and when difficulties of location require that the transit be set up at substations on the curve, then the numerical work, although worked out on precisely the same principle, is much greater and chances for numerical error are greater. The following rule for instrumental work is as simple as any for the simple cases and is far better for the more complicated cases. Compute the deflections for all stations and substations as illustrated above. Set up the transit at the P.C., and locate from it all stations that may be conveniently reached. Then move up the transit to a forward station and use the following rule:

When the transit is set at any forward station, backsight to ANY *previous station with the plates set at the deflection angle*

for the station sighted at. Plunge the telescope and sight at any forward station with the deflection angle computed for that station.

The student should verify for himself the truth of this rule by drawing out a simple case and noting the angles both for foresight and backsight for any station, when the transit is located at any station.

Curve location requires extreme care on the part of field men, for a very slight inaccuracy is apt to be multiplied until the error is intolerable. The transit should be very carefully centered over hubs, which should be referred to points which will not be disturbed during construction.

26. Special Methods of Location. The above method, using a transit and tape, is the ordinary and preferable method, but it is

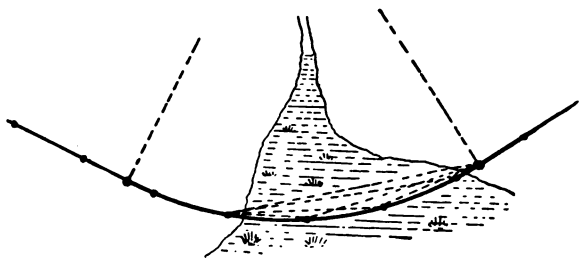


Fig. 9.

sometimes necessary to lay out a curve when a transit is not at hand and there are sometimes special conditions when a modification of the above method will be more accurate. The engineer must have learned the fundamental principles of curve location so thoroughly that he may decide on the best method to use and even to invent some modification which may best suit the special case in hand. A few of these special cases will be described.

(*a*) *Using two transits.* The location may run over swampy ground where accurate chaining is impracticable. Some point of the curve beyond the swamp may be located, perhaps by triangulation, by computing its angle of deflection and the length of the long chord (equation 7). The point beyond the swamp may or may not be the P. T. Then set up two transits simultaneously at the stations located on firm ground The deflection of each chord from

the tangent to the curve at the instrument point, or from the long chord, is a simple matter of geometry (see § 23). A rodman can locate each point by placing himself at points where he is simultaneously in line for both transits.

(*b*) *By tangential offsets.* The solution of this as well as the following methods will be indicated by the lines in the figures. In each case the solution is an application of simple geometrical and trigonometrical principles. The solutions are somewhat lengthened, although not essentially modified, when the curve begins or ends with a subchord. In Fig. 10, for example

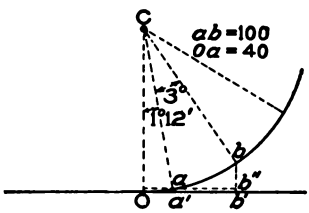


Fig. 10.

$$Ob' = Oa' + a'b' = 40 \cos 0° 36' + 100 \cos (1° 12' + 1° 30')$$

$$bb' = b'b'' + b''b = 40 \sin 0° 36' + 100 \sin (1° 12' + 1° 30')$$

and similarly for other points.

(*c*) *By middle ordinates.* Compute first the length of a long chord for *two* stations and the middle ordinate of such a chord. For subchords, compute the long chord and middle ordinates for an angle *twice* that subtended by the subchord. These distances should be laid off on the ground as indicated in the illustration. In Fig. 11, Oa'' is *half* the long chord for two stations and $a''a$ equals the middle ordinate for such a long chord. Lay off Oa on the tangent and measure out the offset $a''a$. Measure out aa' $(= a''a)$ so that aa' is perpendicular to Oa', and produce Oa' to b. $Oa'' = Oa' = a'b$. Thus is b located, and c, d, etc., will be located similarly. In Fig. 12, an is half the long chord for twice the arc Oa, and On is its middle ordinate. Compute similarly zy and $z''z$, and lay off on the ground a and z. Compute, as in the regular case, aa' and za' $(= a'b)$; b is then laid off as before.

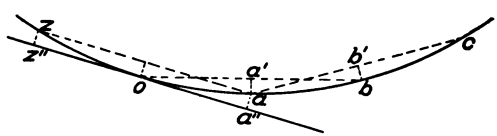


Fig. 11.

(*d*) *By offsets from the long chord.* The geometry involved is apparent from an inspection of Fig. 13, in which is shown the general case of a curve beginning and ending with a subchord. All of the above methods are mathematically perfect in theory, but when curves are thus laid out without the aid of a transit the work is apt to be inaccurate unless unusual care is taken.

27. Obstacles to Location. As in the previous section, the problems are usually simple examples in geometry and trigonometry, and the engineer must select the solution which will give the best result.

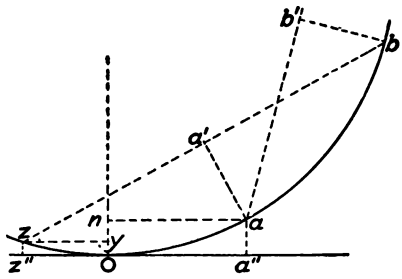


Fig. 12.

(*a*) *Vertex inaccessible.* The tangents are frequently fixed by certain conditions, and yet the intersection of the tangents is within a building or in some place where it is impossible to set up a transit. In the case shown in Fig. 14, the tangents are given by the points a, b, n and m. By measuring the angles baV and abV and the distance ab, the triangle abV may be solved, and the distances aV and bV computed. The external angle at V is the sum of the angles at a and b, and equals the total central angle Δ. Having decided on the radius, the tangent distances are computed by equation 6, and then the differences Bb and Aa can be measured off and the P.C. and the P.T. are thus obtained. As a check on the whole work, the curve, run in by the usual methods, should end exactly at B, with the forward tangent coinciding in direction with Bn.

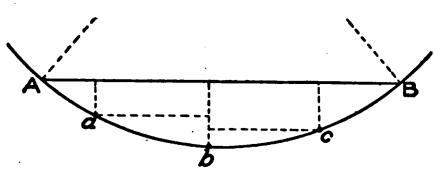


Fig. 13.

(*b*) *Point of curve, or point of tangent, inaccessible.* By making a diagram of the desired line with its obstructions, as in Fig. 15, the known and unknown quantities are readily determined, also their geometrical relations. For example, in the illustration the position of V (on the ground) is known, as is also the distance

AV. Then the *computed* position of A is known. Assume some angle a such that

$$\text{R vers } a = \text{A}s = nv = py$$

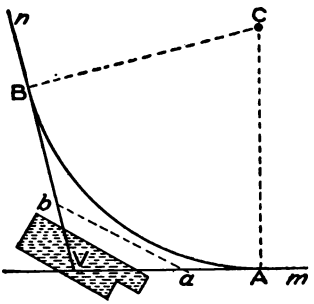


Fig. 14.

where s is in an accessible position. Then

$$ns = sp = \text{R} \sin a$$

and n and p can be located on the ground. Then, setting up a transit at n, and turning from the line np an angle of a, the tangent is determined and the remainder of the curve can be run in as usual. If the P.T. is inaccessible, the curve may be run in to some point m, from which, by similar calculations and field work, the point x is obtained, from which the tangent can be continued.

(*c*) *Middle part of curve obstructed.* The curve may be run as usual to some point n (Fig. 16) which should preferably, although not necessarily, be an even station. At n a chord nm may be run which will clear the obstruction. The angle between nm and the tangent is one-half the angle measured by the arc nm. From equation 7, the length of nm may be computed and then measured off, thus establishing the point m, from which the remainder of the curve is easily run in. As an illustration of the elasticity of this general method, it might under some conditions be easier to run the dotted curve having the same radius as the required curve could then be found by using the same geometrical principles used in §26 *d*.

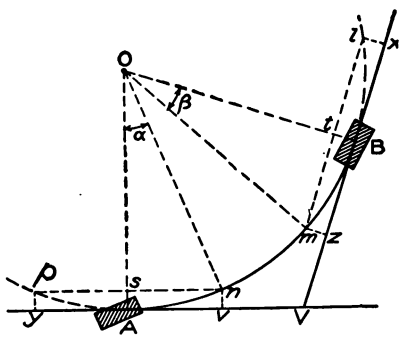


Fig. 15.

28. Numerical Examples. All problems have hitherto been so very simple that nothing has been said about the details of solv-

ing numerical problems. But as problems become more complicated, the greater becomes the value of a systematic method of solution, which may be readily reviewed, checked and studied for the discovery of a possible error. Logarithms should almost invariably be used for multiplication and division, for they are great time-savers. Even if the student is unaccustomed to them, it pays to become familiar with them. Such methods will be used in the following solutions and the student is urged to solve all such problems similarly.

1. In a case similar to that sketched in Fig. 14, ab was measured as 476.25; the angle Vab was measured as 24° 18′, and the angle Vba 34° 22′. The curve is to be a 3° 30′ curve. Its radius is therefore 1637.3. $\Delta = 24° 18' + 34° 22' = 58° 40'$. Compute aA and bB.

		Logarithms.
Equation 6.	R (3° 30′)	3.21412
	$\tan \frac{1}{2}\Delta = \tan$ 29° 20′	9.74969
	T = 920.04	2.96381
$aV = ab \frac{\sin 34° 22'}{\sin 58° 40'}$	ab = 476.25	$2.67778\overline{3}$
	log sin 34° 22′	$9.75165\overline{5}$
	co-log sin 58° 40′	$0.06836\overline{6}$
	aV = 314.74	$2.49795\overline{5}$
	Tan AV = 920.04	
	aA = 605.30	
$bV = ab \frac{\sin 24° 18'}{\sin 58° 40'}$	ab = 476.25	$2.67778\overline{3}$
	log sin 24° 18′	$9.61438\overline{8}$
	co-log sin 58° 40′	$0.06846\overline{6}$
	bV = 229.45	$2.36068\overline{8}$
	Tan BV = 920.04	
	bB = 690.59	

2. Example as in Fig. 15. D = 3° 20′. Δ = 23° 40′. It is estimated that at v, 180 feet back from V, the line np will probably clear the obstruction at A; ns is the difference between 180 and the computed tangent distance AV; $ns \div$ R = sin a. Then $nv = py$ = R vers a. Locate n by the offset vn, and make a

similar offset at y. If this line does not clear the obstruction, another value of a (probably greater) should be assumed and new values for Av and vn computed. Compute the numerical values as above.

29. Modifications of Location. Only a few of the very many changes which are at times required will here be given. They are all solvable by a few principles of geometry and trigonometry. The occasion for many such changes is the adjustment of the inaccuracies of a "paper location."

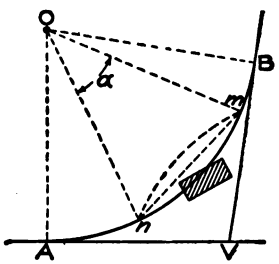


Fig. 16.

(1) *To move the forward tangent parallel to itself a distance x, the radius remaining unchanged.* See Fig. 17. Every point of the curve is moved parallel to the first tangent a distance $AA' = BB' = VV' = OO'$.

$$AA' = \frac{B'n}{\sin nBB'} = \frac{x}{\sin \Delta} \qquad (11)$$

(2) *To move the forward tangent parallel to itself, the point of curvature remaining unchanged.* Since the central angle (Δ) is unchanged, the curve and all its parts are simply enlarged or reduced according to some ratio, as is apparent from Fig. 18. The known quantities are the change in the tangent x' (or x''), the central angle Δ and the original radius R.

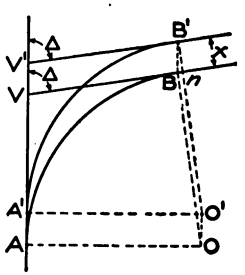


Fig. 17.

$$VV' = \frac{V'h}{\sin hVV'} = \frac{x'}{\sin \Delta} \qquad (12)$$

Then the new tangent distance $AV' = AV + VV'$. The triangle $Bm B'$, being similar to the triangle $AO'B'$, is isosceles and $Bm = B'm$. Then the new radius

$$R' = R + mB = R + \frac{B'r}{\text{vers } B'mB} = R + \frac{x'}{\text{vers}\,\Delta} \qquad (13)$$

The modifications of this solution, when the tangent is moved toward the center, are very simple and are apparent from the figure.

(3) *To change the direction of the forward tangent at the point of tangency.* The central angle scaled off from the paper location might have an error which would be best corrected by this means. This solution is but one of a large class in which the central angle is modified. The required change (a) in the central angle is one of the given quantities. R, Δ, AV and BV are also known. In Fig. 19, $\Delta' = \Delta - a$; $Bs = R \text{ vers } \Delta$; $Bs = R' \text{ vers } \Delta'$

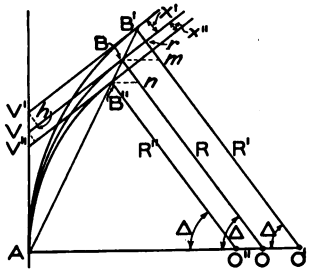


Fig. 18.

$$\therefore \quad R' = R \frac{\text{vers } \Delta}{\text{vers } (\Delta - a)} \qquad (14)$$

Also, since $As = R \sin \Delta$ and $A's = R' \sin \Delta'$, we have

$$AA' = A's - As = R' \sin \Delta' - R \sin \Delta \qquad (15)$$

30. Examples. 1. Given a 4° 20′ curve with a central angle of 18° 28′. It is required to move the forward tangent parallel to itself 12 feet. How much is the change of the P.C. (the distance AA′ in Fig. 17)?

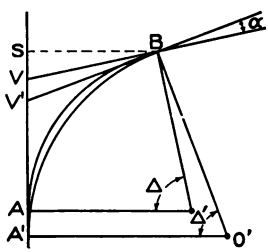


Fig. 19.

2. Given the same curve as above, it is required to move the tangent *toward* the center 12 feet, but without changing the P.C. What will be the changes in the tangent distance and the radius?

3. Given the same curve as above, it is required to *diminish* the central angle by 0° 22′, but retaining the same P.T. What will be the new radius and the change in the P.C.?

COMPOUND CURVES.

31. Definition. Compound curves consist of a succession of two or more curves of different radii which have a common tangent where they meet. They *may* be laid out by the same method

as simple curves, but there are certain geometrical relations existing between the parts of a compound curve which greatly facilitate the computations, especially when any modifications are required. In the following demonstrations R_1 and R_2 will always represent the smaller and larger radii respectively, no matter which succeeds the other. Δ_1 and Δ_2 will always represent the corresponding central angles. Although R_2 is always larger than R_1, Δ_2 may or may not be larger than Δ_1. T_2 is always adjacent to the larger radius R_2 and is always larger than T_1.

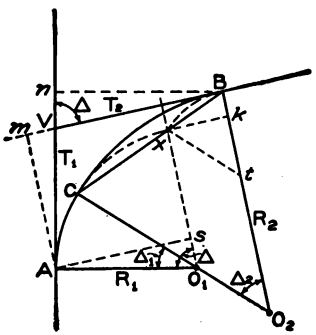


Fig. 20.

32. Mutual Relations of the Parts of a Compound Curve of Two Branches. The curve is illustrated in Fig. 20, in which AC and CB are the two curves with radii of R_1 and R_2 respectively. Therefore by the above definitions the other functions are as indicated in the figure. Produce the arc AC until the angle $CO_1 x = \Delta_2$. Then, by similar triangles, the chord Cx produced must intersect B. Also, if xt is drawn parallel to CO_2, it will equal Bt and the angle xtB will equal Δ_2. Then draw As and xk perpendicular to $O_1 x$. Then

$$Bk = xt \text{ vers } xtB = (R_2 - R_1) \text{ vers } \Delta_2$$
$$xs = AO_1 \text{ vers } AO_1x = R_1 \text{ vers } \Delta$$
$$Am = AV \sin AVm = T_1 \sin \Delta$$
$$Am = Bk + xs$$
$$\therefore T_1 \sin \Delta = R_1 \text{ vers } \Delta + (R_2 - R_1) \text{ vers } \Delta_2 \quad (16)$$

By drawing a few additional lines in the figure, it may similarly be proved that

$$T_2 \sin \Delta = R_2 \text{ vers } \Delta - (R_2 - R_1) \text{ vers } \Delta_1 \quad (17)$$

By algebraic transformation we may derive from equations 16 and 17 the following useful relations. The details of the derivation of these equations is suggested as a profitable exercise for the student.

$$R_2 = R_1 + \frac{T_1 \sin \Delta - R_1 \text{ vers } \Delta}{\text{vers } (\Delta - \Delta_1)} \quad (18)$$

$$R_2 = \frac{T_1 \sin \Delta \operatorname{vers} \Delta_1 - T_2 \sin \Delta (\operatorname{vers} \Delta - \operatorname{vers} \Delta_2)}{\operatorname{vers} \Delta_2 \operatorname{vers} \Delta_1 - (\operatorname{vers} \Delta - \operatorname{vers} \Delta_1)(\operatorname{vers} \Delta - \operatorname{vers} \Delta_2)} \quad (19)$$

33. Modifications of Location. As in § 29, only a few of the most common modifications will be here illustrated.

1. To move the forward tangent parallel to itself, but without changing the radii. From Fig. 21 we derive

$x = O_2s - O_2's' = (R_2 - R_1) \cos \Delta_2 - (R_2 - R_1) \cos \Delta_2'$, from which

$$\cos \Delta_2' = \cos \Delta_2 - \frac{x}{R_2 - R_1} \quad (20)$$

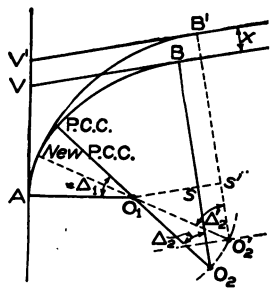


Fig. 21.

Fig. 21 shows the tangent as having been moved *outward*; in such a case the P.C.C. (which means the "point of compound curvature") is moved *backward* along the sharper curve. If it is desired to move the tangent toward the center, the required equation may be found by transposing Δ_2 and Δ_2' in equation 20. But in this case the sharper curve must be extended and the P.C.C. must be moved *forward*.

In case the larger radius comes first, the figure is apparently quite different, although a little study will show that the same principles apply. From Fig. 22 we derive

$x = O_1's' - O_1s = (R_2 - R_1) \cos \Delta_2' - (R_2 - R_1) \cos \Delta_1$ from which we have

$$\cos \Delta_1' = \cos \Delta_1 + \frac{x}{R_2 - R_1} \quad (21)$$

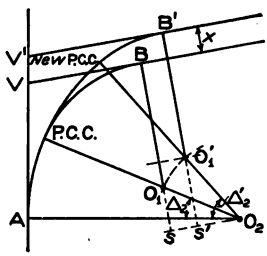


Fig. 22.

Fig. 22 shows the tangent as having been moved *outward*; in such a case the P.C.C. is moved *forward* along the easier curve which is extended. As before, if the tangent is moved *inward*, transpose Δ_1 and Δ_1' in equation. The P.C.C. will then be moved *backward* along the first curve.

(2) To change the radius of one of the curves without changing either tangent. The requirements will be apparent from a "paper" solution. In Fig. 23 assume that the longer

radius, which comes first, is to be shortened by an amount equal to sO_2. The new center O′ must lie somewhere on the arc whose center is O_1 and whose radius is O_1s. It also must lie on a line which is parallel to AV and distant from it by R_2' which is equal to s – P.C.C. With O_2' as center, draw an arc from O_1 to m'. With O_2 as center, draw an arc from O_1 to m. It may be proved that mm' is parallel to AV. Draw the line O_1n perpendicular to AO_2.

$$mn = (R_2 - R_1) \text{ vers } \Delta_2;\ m'n' = (R_2' - R_1) \text{ vers } \Delta_2';\ mn = m'n'.$$

$$\text{vers } \Delta_2' = \frac{(R_2 - R_1)}{(R_2' - R_1)} \text{vers } \Delta_2 \qquad (22)$$

$$AA' = O_1n - O_1n' = (R_2 - R_1) \sin \Delta_2 - (R_2' - R_1) \sin \Delta_2' \qquad (23)$$

34. Examples. 1. A 5° 30′ curve with a central angle of 16° 22′ has a tangent distance of 1800 feet from the P.C. to the vertex. At the P.C.C. the curve compounds into an easier curve. The total central angle is 30° 18′. What is the radius of the easier curve, and what is tangent distance?

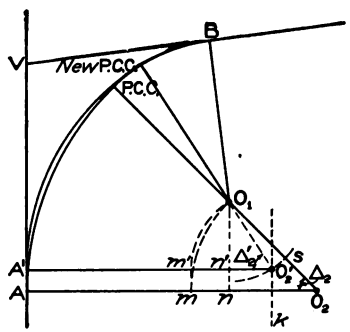


Fig. 23.

Answer. The given quantities are R_1 the *shorter* radius, Δ_1, Δ, and T_1; the required quantities are R_2 and T_2. By substituting the known quantities in equation 18 and then the computed valve of R_2 in equation 17, the required quantities are found. The student should perform this numerical work.

2. A 2° 30′ curve 450 feet long runs into a 5° 30′ curve 260 feet long. It is required to move the forward tangent *in* 6.4 feet, but without changing either radius. Required the change in the P.C.C. *Comment.* The solution evidently requires the indicated modification of equation 21. It should be noted that a practical solution always requires that the resulting value of the cosine shall be less than unity, which means that x can *never* be greater than $(R_2 - R_1)$, and also means that the sum of the cosines of the

original angle and its modified angle shall be less than unity. The linear change in the P.C.C. may be found by the formula

$$\text{Linear change} = (\text{angular change in degrees}) \times (\text{radius of curve}) \times \frac{\pi}{180^\circ}$$

3. Assume the same curve as in example 2, but that it is required to change the 2° 30′ curve to a 2° curve without changing the tangents. *Comment.* Fig. 23 may be made to apply by transposing the new and old larger radii, and their corresponding central angles. Note that when such changes are made in equation 22, the equation is unchanged. The effect on equation 23 is merely to change the algebraic sign, which means that the P.C.C. is moved *backward* instead of forward.

4. Draw a figure corresponding to Fig. 23 but showing a change in the smaller radius R_1.

TRANSITION CURVES.

35. Transition Curve Systems. *General Use.* When a train, or any other mass, is in motion it requires a definite force to make it move in a curved path. If the two rails of a railroad curve are level transversely, this centripetal force can only be furnished by the pressure of the wheel flanges against the outer rail. To avoid such a dangerous pressure, which would make high speed impracticable, the outer rail is made higher than the other. But it is manifestly impracticable to suddenly raise the outer rail at the beginning of a curve and lower it as suddenly at the end of the curve. There must be a "run-off" of considerable length. If this run-off were placed exclusively on the tangent, there would be an objectionable jar because the cars were tipped on a straight track where there is no compensating centrifugal force. If the run-off were entirely on the curve there would be a jar, because the centripetal force would not become fully developed at the beginning of the curve; and, therefore, a transition curve is used at the beginning and end of the curve. The transition curve is one whose rate of curvature gradually increases from nothing to the rate of the central part of the curve. If the super-elevation of the outer rail is begun at the beginning of the transition curve and is grad-

ually increased as the curvature increases so that the proper super-elevation is attained at the end of the transition curve where the regular curve commences, then the theoretical requirements are satisfied. But there is another important reason for transition curves. On a curve the bogie trucks of a car make an angle with the axis of the car. If there were an instantaneous change from a straight track to the full degree of curvature the change of position of the truck would need to be accomplished in the time required for its train to run the distance between the truck centers of a car. For a high-speed train this distance would be covered in less than a second. On a transition curve this change of position is accomplished gradually and without jar. The amount of the required super-elevation will be discussed in the following sections.

Varieties of Curves. A theoretically exact transition curve is very complicated and its mathematical solution very difficult. Many systems of curves have been proposed, all of which are objectionable for some one of the following reasons, as stated in a report by a Committee of the American Railway Engineering Association. "(1) If simple approximate formulas were used they were not sufficiently accurate. (2) Accurate formulas were too complex. (3) The curve could not be expressed by formulas. (4) Formulas were of the endless series class. (5) Complex field methods were required to make the field work agree with formulas with spirals of large central angles." The Committee then developed a method which gives results whose accuracy is beyond that of the most careful field work and yet which is sufficiently simple for practical use. The mathematical development is too elaborate to be detailed here but the working formulas, together with a condensation of the Tables will be given, together with an explanation of their practical use and application, with examples.

The general form of these curves, whatever their precise mathematical character, is shown in Fig. 24. AVB are two tangents, joined by the simple circular curve ACB, having the center O. Assume that the entire curve is moved in the direction CO a distance $OO' = CC' = BB' = AA'$. At some point TS on the tangent, the spiral begins and joins the circular curve tangentially at SC. The other spiral runs from CS to ST. The significance of these symbols

may be readily remembered from the letters; T, S, and C signify tangent, spiral, and circular curve; TS is the point of change from tangent to spiral, SC the point of change from spiral to curve, etc. At the other end of the circular curve, the letters are in the reverse order, the station numbers increasing from A to B. The meaning of each of the various symbols used is plainly indicated on the diagram, Fig. 24.

The *length* of a spiral can only be computed on the basis of certain assumptions as to the desired rate of tipping the car, so as to avoid discomfort to passengers, and of course this depends on the expected velocity. There is also a limitation since the sum of the two spiral angles cannot exceed the total central angle of the curve. The *minimum* lengths recommended are as follows:

On curves which limit the speed:

6° and over, 240 feet
less than 6°, $5\frac{1}{3}\times$speed in m.p.h. for elevation of 8 inches

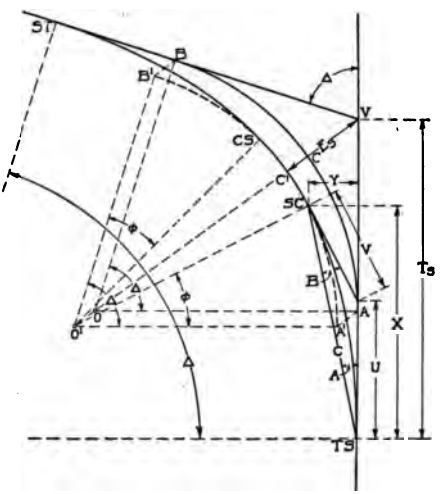


Fig. 24.

On curves which do not limit the speed:

30 times elevation in inches, OR
$\frac{2}{3}\times$ultimate speed in m.p.h. $\times$ elevation in inches

For example. (1) 5° curve which limits speed; speed limit 48 m.p.h. by interpolation in table, § 41; $48\times5\frac{1}{3}=256$ feet minimum length. (2) 3° curve; maximum operating speed 60 m.p.h.; superelevation .62 feet = 7.44 inches; $30\times7.44=$ 223.2 feet; OR, $\frac{2}{3}\times60\times7.44=297.6$ feet. Of course the higher value should be used, or say 300 feet as the minimum length. While it is generally true that the longer transition curves give easier riding, the spiral must

not reach the center point of the curve. Since it is approximately true that the spiral extends for equal distances on each side of the original point of curve, it is nearly true that two spirals, each having the same length as the original curve, would meet at the center. The length of a spiral should in general be far less than the length of the original curve.

36. Symbols. Besides the symbols whose significance is clearly indicated in Fig. 24, the following are defined:

a—The angle between the tangent at the TS and the chord from the TS to any point on the spiral.

A—The angle between the tangent at the TS and the chord from the TS to the SC.

b—The angle at any point on the spiral between the tangent at that point and the chord from the TS.

B—The angle at the SC between the chord from the TS and the tangent at the SC.

D—The degree of the central circular curve.

Δ—The central angle of the original circular curve, or the angle between the tangents.

ϕ—The central angle of the whole spiral.

ϕ_1—The central angle of the spiral from the TS to the first spiral point.

k—The increase in the degree of the curve per station on the spiral.

L—The length of the spiral expressed in feet from the TS to the SC.

s—The length of the spiral in stations from the TS to any given point.

S—The length of the spiral expressed in stations from the TS to the SC.

37. Deflections. The field formulas for deflections are based on the following two equations

$$a = 10ks^2 \text{ minutes} = \frac{1}{3}\phi_1$$

$$a = 10k\mathrm{S}^2 \text{ minutes} = \frac{1}{3}\phi$$

The first deflection $a_1 = 10ks_1^2$ minutes. But k is the increase in

degree of curve per station and since the degree of curve increases as the length $k = D \div S$, S being expressed in stations. For point 1, since $S = 10s$, $a_1 = 10\left(\frac{D}{10s}\right)s^2 = Ds$, which may be expressed as the degree of curve times the length of the chord in stations. For example, if the spiral is 400 feet long ($L = 400$ and $S = 4$) and runs on to a 5° curve ($D = 5$), one chord is 40 feet long or $s = .4$ stations. Then $a_1 = 5 \times 0.4 = 2$ minutes of arc for the deflection for the first chord point. And since the deflections are as the square of the number of stations, the deflections from TS to succeeding stations will be 4, 9, 16, 25, 36, 49, 64, 81, and 100 times 2 minutes, as shown in the second vertical column of Part A of Table IV. The last deflection $A = 100 \times 2' = 200' = 3°\ 20' = \frac{1}{3}\ (10°) = \frac{1}{3}\phi$, which is the total central angle of the spiral. This result also checks the general equation

$$\phi = \frac{kS^2}{2} = \frac{DS}{2} = \frac{kL^2}{20000} = \frac{DL}{200}$$

Since

$$\phi = \frac{DS}{2} = \frac{5 \times 4}{2} = 10°$$

The deflections from any point of the spiral to any other point, either forward or backward, may be found by multiplying the value of a_1 (in this case 2′) by the coefficients in the proper vertical column of that table. The values of the ratios $\frac{U}{L}$ and $\frac{V}{L}$ for even degrees and for A, $\frac{C}{L}$, $\frac{X}{L}$, and $\frac{Y}{L}$ for half degrees are given in Parts B and C, Table IV.

38. Insertion of a Spiral between a Tangent and a Circular Curve. In Fig. 25 it has been necessary to make the distance MM′ about 100 times its real proportional value in order to make the illustration distinct. The curve AMB is a simple circular curve joining the two tangents, such as would be used without spirals. If a suitable spiral is started at a suitable point Q, and run to some point Z, such that the total central angle of the spiral, ϕ, equals the angle ZO′N of the curve, and then the circular curve having a common tangent with the spiral at Z, is run to Z′, from

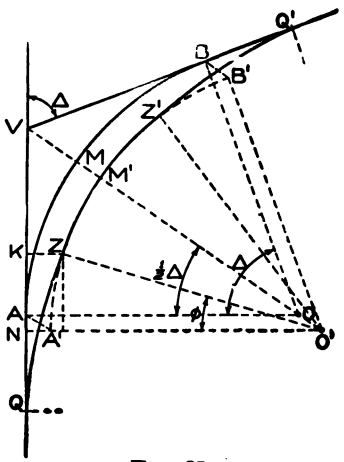


Fig. 25.

which a similar spiral is run to the tangent at Q', such a combination of curves will give the desired result. It is now necessary to compute the dimensions required to accomplish this result. The introduction of the spiral has the effect of throwing the curve away from the vertex by the amount MM', which we will call m; it also reduces the central angle of the circular curve by 2ϕ. ZK is the y ordinate of the chosen spiral and QK the x ordinate.

$$A'N = y - R \text{ vers } \phi.$$

Therefore,

$$m = MM' = AA' = BB' = \frac{A'N}{\cos \frac{1}{2}\Delta} = \frac{y - R \text{ vers } \phi}{\cos \frac{1}{2}\Delta} \qquad (24)$$

$$NA = AA' \sin \frac{1}{2}\Delta = (y - R \text{ vers } \phi) \tan \frac{1}{2}\Delta$$

$$VQ = QK - KN + NA + AV$$

$$= x - R \sin \phi + (y - R \text{ vers } \phi) \tan \frac{1}{2}\Delta + R \tan \frac{1}{2}\Delta$$

$$= x - R \sin \phi + y \tan \frac{1}{2}\Delta + R \cos \phi \tan \frac{1}{2}\Delta \qquad (25)$$

As a numerical solution of any problem will usually involve the determination of the value of A'N, equation 25 may be reduced from four terms to three as follows: Transform the equation above equation 25 to read

$$VQ = x + R (\tan \frac{1}{2}\Delta - \sin \phi) + A'N \tan \frac{1}{2}\Delta \qquad (26)$$

$$VM' = VM + MM' = R \text{ exsec } \frac{1}{2}\Delta + \frac{y}{\cos \frac{1}{2}\Delta} - \frac{R \text{ vers } \phi}{\cos \frac{1}{2}\Delta} \qquad (27)$$

$$AQ = VQ - AV = x - R \sin \phi + (y - R \text{ vers } \phi) \tan \frac{1}{2}\Delta \qquad (28)$$

39. Example. It is required to join two tangents which make an angle of 28° 16′ by a 6° curve terminated by suitable spirals. Assume a 10-chord spiral 240 feet long. Then

$$\phi = \frac{DL}{200} = \frac{6 \times 240}{200} = 7.2° = 7° 12'$$

From Part C, Table IV, when $\phi = 7.2°$

$$\frac{X}{L} = .998517 - \frac{2}{5}(.998517 - .998298) = .998430$$

$$X = .998430 \times 240 = 239.623$$

$$\frac{Y}{L} = .040681 + \frac{2}{5}(.043581 - .040681) = .041841$$

$$Y = .041841 \times 240 = 10.042$$

$$\frac{1}{2}\Delta = 14° 8'$$

			Logarithms
(Equation 24)		R	2.98017
	Y 10.042	vers 7° 12′	7.89682
	7.533		0.87699
	A′N 2.509	$\cos \frac{1}{2}\Delta$	0.39950
			9.98665
	$m = MM' = AA' = 2.587$		0.41285
(Equation 27)		R	2.98017
		exsec $\frac{1}{2}\Delta$	8.49436
	VM = 29.821		1.47453
	m = 2.587		
	VM′ = 32.408		
(Equation 26)	nat. tan. $\frac{1}{2}\Delta$ = 0.25180		
	nat. sin ϕ = 0.12533		
	0.12647		9.10198
x = 239.623		R	2.98017
120.825			2.08215
	(see above)	A′N	0.39950
		tan $\frac{1}{2}\Delta$	9.40106
0.632		AN	9.80056
VQ = 361.080		R	2.98017
(Equation 28)		tan $\frac{1}{2}\Delta$	9.40106
240.564		AV	2.38123
AQ = 120.516			

40. Insertion of Spirals in Old Track. An engineer frequently has occasion to insert spirals in track which was not so laid out. The simplest method from a mathematical standpoint is that given in the two previous sections. But this would involve moving the whole track for the entire length of the curve, and also, since it is apparent from Fig. 25 that the new line QZZ'Q' is shorter than the old line QAMBQ', the method will involve rail cutting and the boring of holes for track bolts. The following method makes a slight sharpening of curvature of the middle circular section, which at the center is slightly *outside* of the old track, and so crosses the old line that the lateral deviation from the old line is always very small and the length of the new track need not differ appreciably from that of the old. The method of solution is indicated in Fig. 26.

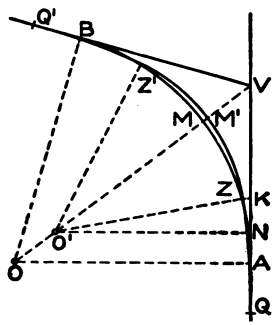


Fig. 26.

$O'N = R' \cos \phi + y$ (This shows more clearly in Fig. 25.)

$$O'V = O'N \sec \frac{1}{2}\Delta$$

$$= R' \cos \phi \sec \frac{1}{2}\Delta + y \sec \frac{1}{2}\Delta$$

$$m = MM' = MV - M'V$$

$$= R \operatorname{exsec} \frac{1}{2}\Delta - R' \cos \phi \sec \frac{1}{2}\Delta - y \sec \frac{1}{2}\Delta + R' \qquad \textbf{(29)}$$

$$AQ = QK - KN + NV - VA$$

$$= x - R' \sin \phi + (R' \cos \phi + y) \tan \frac{1}{2}\Delta - R \tan \frac{1}{2}\Delta$$

$$= x - R' \sin \phi + R' \cos \phi \tan \frac{1}{2}\Delta - (R - y) \tan \frac{1}{2}\Delta \qquad \textbf{(30)}$$

$$\text{The length of the old line from Q to Q'} = 2AQ + 100 \frac{\Delta}{D}. \qquad \textbf{(31)}$$

$$\text{The length of the new line from Q to Q'} = 2L + 100 \frac{\Delta - 2\phi}{D'},$$

in which L is the length of each spiral.

41. Example. Assume that a track has been laid with a 6° curve for 39° 50′: It is desired to insert suitable spirals without altering the length of the old track. *Solution.* Unfortunately there is no method of solving this problem so as to obtain directly the revised value for the radius: The new radius will always be about 5 per cent shorter than the old. The larger the central angle, the less will be the difference. The only method is therefore to assume a value for R′, solve equation 30, and then compare the lengths of the new and old lines. If the difference is so small that it may be neglected, the problem is solved. If not, a slight modification of the new radius, such as experience in these computations will suggest, will usually give on a second trial a value which is sufficiently close. As a first trial for the above numerical case, we will assume a 6° 20′ curve for the new curve, and a 240 ft. spiral whose $\phi = 7° 36'$. $x = 239.580$ and $y = 10.60$.

			Logarithms.
$x = 239.580$		R′ (6° 20′)	2.95671
		sin 7° 36′	9.12141
	119.709		2.07812
		R′	2.95671
		cos 7° 36′	9.99617
		$\tan \frac{1}{2}\Delta$	9.55909
325.069			2.51197
		R = 955.37	
		$y =$ 10.60	
		944.77	2.97532
		$\tan \frac{1}{2}\Delta$	9.55909
	342.310		2.53442
564.649	462.019		
462.019			
AQ = 102.630			

The length of the old curve from Q to Q′ is

$100 \frac{\Delta}{D} = 100 \frac{39.83333}{6} =$	663.889
$2\,AQ = 2 \times 102.630 =$	205.260
	869.149

The length of the new curve from Q to Q′ is

$$100\frac{\Delta-2\phi}{D'}=100\frac{39.8333°-15.2°}{6.333°}= \qquad 388.947$$

$$2L = 2 \times 240 = \qquad 480.000$$

$$868.947 \quad 868.947$$

Difference in length = 0.202

When it is considered that in that length of 869 feet there will be about 27 rail joints and that a stretching at each joint of about .0075 foot will make up for this difference of length, it might not be necessary to cut the rails.

To illustrate the method of adjustment if a more accurate value for R′ were required; note that in the above case the new curve is too short. If R′ is diminished (say from D′ = 6° 20′ to D = 6° 24′), one term of equation 30 will be increased and one of them diminished, but the net change in the value of AQ is 3.403, which will decrease the length of the old curve by 6.806. But such a change in D′ will decrease the length of the new line by 6.552.

The revised length of the old line is, therefore, 862.343 and the revised length of the new line is 862.395

The revised difference is 0.052

The new line is now longer than the old, but the difference is insignificant (about one fortieth of an inch per joint). By interpolation D′ = 6° 23′ is the better value to use.

There is another method of introducing a spiral into old track without even changing the central part of the curve. The spiral runs into a curve which is sharper than the original curve which then compounds into the old curve. The solution of this method consists in so choosing and locating the spiral and the sharper curve that it will compound into the original curve. The details of this method will not be here given because, although it involves less track work at the start, it is a more complicated alignment to maintain and the method is inferior to the one previously given.

On the basis of $D' = 6° 20'$

			Logarithms.
(Equation 29)		R (6°)	2.98017
		exsec $\frac{1}{2}\Delta$	$\bar{8}.80356$
	60.776		$\bar{1}.78373$
	$R' = 905.13$		
	965.906	R′	2.95671
		cos ϕ	9.99617
		sec $\frac{1}{2}\Delta$	$\bar{0}.02678$
	954.255		$\bar{2}.97966$
		log $x =$	$\bar{1}.02530$
		sec $\frac{1}{2}\Delta$	$\bar{0}.02678$
	11.274		1.05209
	965.529		
	$m =$ 0.377 foot		

Note that the maximum lateral change in the track is less than five inches.

On many railroads where there has been no pretense to using spirals the track foremen have produced nearly the same result in a rough uncertain way by throwing in the curve somewhat near the point of curve. This necessarily sharpens the curve further on, and thus substantially the same result is obtained as is described above but without any calculations or any theoretical accuracy. It is only by such means that a tolerable riding track can be produced when transition curves are not used.

42. Use of Transition Curves with Compound Curves. It is shown in the last few sections that the lateral deviation involved by the use of spirals is very small. Since compound curves are usually employed only when the location is difficult, it is best to make the calculations as if no spirals were to be used, except that approximate allowances may be made for such lateral changes as will be required. Then the changes can be computed as follows. Theoretically there should be a transition curve between the two branches of a compound curve, but when a train is already on a curve and the wheels are pressing against the outer rail, it will cause but little jar to merely increase the curvature. The intro-

duction of such spirals will not be here given. Transition curves need not be used in running on to curves which are easier than 3° and even 4°. Therefore if one branch of a compound curve is of easy curvature, as is frequently the case, it will not be necessary to use a spiral at that end of the curve. Therefore two cases will be developed—using spirals at one end only, and at both ends.

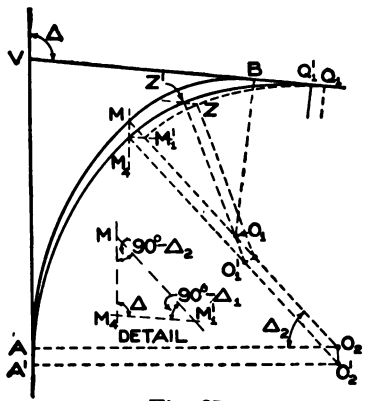


Fig. 27.

(*a*) *Spiral at one end only.* The method of §38 may be adopted by substituting Δ_1 for $\frac{1}{2}\Delta$ in equations 24 to 28. This would move the P.C.C. from M to M'. But since the two curves *must* be made to coincide, the sharper curve will be moved parallel to the tangent BV a distance of M'M, while the unchanged circular curve will be moved parallel to the tangent AV a distance MM_4. Calling $MM' = m_1$, we have, Fig. 27,

$$M_1'M_4 = MM_1' \frac{\sin M_1'MM_4}{\sin M_1'M_4M} = m_1 \frac{\sin (90° - \Delta_2)}{\sin \Delta} = m_1 \frac{\cos \Delta_2}{\sin \Delta} \quad (32)$$

$$MM_4 = m_1 \frac{\cos \Delta_1}{\sin \Delta} \quad (33)$$

It should be noted that the new P.C. is at A' and that $AA' = MM^4$, while the P.T. is changed from B to Q_1', which equals $BQ_1 - QQ'$. BQ_1 is found from equation 28 and $Q_1'Q_1 = M_1'M_4$ is found from equation 32.

(*b*) *Spiral at both ends.* Applying the method of §38 to each branch of the curve in turn by successively substituting Δ_1 and Δ_2 for $\frac{1}{2}\Delta$, we will obtain values for m_1 and m_2 which will in general differ considerably. But as before we may move each revised curve as shown in Fig. 28 and as computed in equations 34 and 35. Calling $MM_1' = m_1$ and $MM_2' = m_2$, and noting that the angle at M_1' (see the detail) $= 90° - \Delta_1$, the angle at M_2'

$= 90° - \Delta_2$ and the angle at $M_3 = \Delta$, we have the following:

$$M_1'M_3 = M_1'M_2' \frac{\sin (90° - \Delta_2)}{\sin \Delta} = (m_1 - m_2) \frac{\cos \Delta_2}{\sin \Delta} \quad (34)$$

$$M_2'M_3 = M_1'M_2' \frac{\sin (90° - \Delta_1)}{\sin \Delta} = (m_1 - m_2) \frac{\cos \Delta_1}{\sin \Delta} \quad (35)$$

As in the previous case, the position of the new point of the spiral is found in each case from one of the above quantities and the change computed from equation 28. Note that in one case the point of spiral is moved nearer to, and in the other case further away from, the vertex.

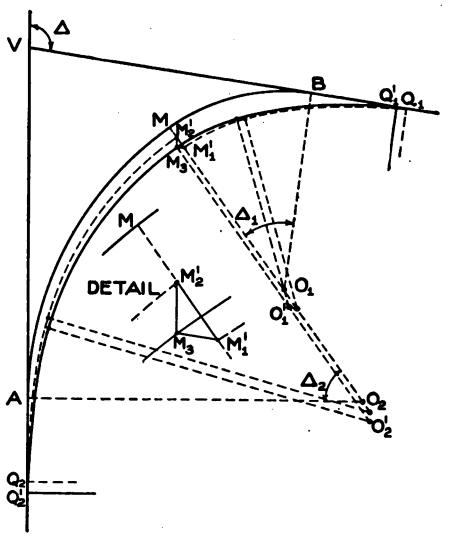


Fig. 28.

43. Example. Given a 7° 20′ curve for 29° 40′ followed by a 4° 10′ curve for 25° 20′. Required to introduce suitable spirals. Using equation 24 successively with the two sets of figures we obtain $m_1 = 5.506$ and $m_2 = 1.930$. Then $(m_1 - m_2) = 3.576$. Substituting this value in equations 34 and 35 we obtain $M_1M_3 = 3.946$ and $M_2'M_3 = 3.793$. Using equation 28, calling $\frac{1}{2}\Delta =$ 25° 20′, we compute $AQ_2 = 120.769$. But we must *add* to this an amount equal to $M_2'M_3 = 3.793$, which makes $AQ_2' = 124.562$. Similarly, calling $\frac{1}{2}\Delta = 29° 40'$ in equation 28, we may compute $BQ_1 =$ 152.444. But from this must be *subtracted* $M_1'M_3 = 3.946$, which makes $BQ_1' = 148.498$. The actual lateral change from the original point M is equal to $MM_2' + M_2'M_3 \sin \Delta_2 = 1.930 + 1.623 = 3.553$. The student should verify in detail all these calculations.

44. Field Work. When spirals form part of the original location, it is a useless refinement to locate all the chord points before earth work has been done. It is then sufficient to locate the beginning and end of each spiral with perhaps one intermediate point in case the spiral is very long. During the resurvey which immediately precedes track work, when the roadbed is graded and flat, the intermediate points are readily inserted. Referring to Fig. 25, the point Q (or TS) would first be located at a distance VQ (see equation 25) from the point V. Assume, as in Fig. 25, a simple curve, 6°, with $\Delta = 32°$, and at each end a spiral 240 feet long. During the first location of the road it would be sufficient to locate the end of the spiral at point Z or SC. The deflection from the tangent when the instrument is at TS and is "sighting at" SC is $\frac{1}{3}\phi = \frac{1}{3}\left(\frac{DL}{200}\right) = \frac{1}{3}\left(\frac{6\times 240}{200}\right) = 2°.4 = 2°24'$. The ordinate X (QK in Fig. 25) is 239.623 and the distance out from the tangent $KZ = y = 10.042$. The total central angle to this point $(\phi) = 7°\ 24'$. The central angle left between Z and $Z' = 32° - (2\times 7°\ 24') = 17°\ 12'$. Set up the instrument at Z. The deflection from the tangent at the point occupied when the instrument is at Z (which is here SC) and is sighting at Q or TS is $B = \frac{2}{3}\phi = 4°\ 48'$. Setting off that deflection on the transit so that when the instrument is turned around to the tangent it shall read 0°, the remaining central angle of 17° 12′ can be laid off in the usual manner. Again setting up the instrument at Z′ (which is point CS of that spiral) the point Q′ or ST is located with the same deflection (4° 48′) as the back sight from Z to Q. The distance from Z′ to the tangent VQ′ is likewise the same as $ZK = 10.042$.

When the intermediate points are to be located, the transit is set up at Q and the points are located by chord lengths of 24 feet and with deflections to the various points as given by multiplying a_1 (which $= 6\times .24 = 1.44$ minutes of arc) by the factors in the column under TS. When the circular curve ZZ′ has been located and the transit set up at Z′ and oriented so that it will read 0° when sighting along the tangent to the curve at Z′, then (using the deflection factors in the column under SC) the deflection to the point 9, 24 feet away, is $29\times 1.44' = 41.76'$; to the next point 8 it is $56\times$

$1.44' = 80.64' = 1°\ 20.64'$; the points 7, 6, etc. are found similarly; to the point of tangency Q it is $200 \times 1.44' = 288' = 4°\ 48'$, as given before. Then the transit should be set up at Q′ and back-sighted on Z′ with a reading of 2° 24′. If the vertex of the curve V had already been previously located and the field work is accurate, the sighting on V should be 0°. Also the reading on any other point of the spiral may be computed from the column of coefficients under TS.

It sometimes becomes necessary to set up the transit at some intermediate point of the spiral, as the point 3. With the instrument set up at 3, use the coefficients under the column 3 in the table, orienting the transit by a sighting at any known point with the transit set at the given deflection for that point. Then, when the telescope is turned to 0°, the transit will be sighting along the tangent to the curve at the point 3 and the deflection to any other point, forward or backward, will be as given in that column by the coefficient times a_1.

It may be noted that the deflections are given to fractions of a minute of arc, which is of course very much closer than an ordinary transit can be used. But the location of spirals demands the closest work attainable with the ordinary field transit; and even though the transit is only graduated to single minutes, a fraction of a minute can be estimated when setting the vernier of a good transit and therefore the precise angles are given so that the closest attainable value of the true angle may be set off.

VERTICAL CURVES.

45. Reasons for Use. Although the change of direction of motion due to a change of grade is never as great as that of an ordinary horizontal curve, yet it is as necessary in one case as in the other to join the two grade lines by a "vertical curve." As in the case of horizontal curves, there is nothing which absolutely determines the rate of curvature. When the grades intersect over a summit a comparatively short curve is permissible, but when passing through a sag the upward bend of the track acts as a literal obstruction and therefore a much longer curve is necessary. Some roads adopt some uniform distance, such as 200 feet each side of the vertex, as the length of all such curves, regardless of the change in the rate of grade. A more logical method is to

make the length a function of the change of grade. A very common rule is to make the length 100 feet for each tenth of one per cent of change of grade. For example, a one per cent grade descending into a hollow is followed by a 1.2 per cent grade climbing out of it; what should be the length of the vertical curve at the sag? The *change* of grade is the numerical sum of the grades or 2.2 per cent; therefore, by the above rule the length should be 2,200 feet. Such a length is excessive, but such a change of grade is also somewhat unusual and hardly to be found where the speed was high. For lower speed such a long vertical curve is not essential.

46. Geometrical Form of the Curve. The method of laying out such a curve is illustrated in Fig. 29, in which the grades are exaggerated enormously. The curve begins and ends at equal distances on each side of the vertex B, as at A and C. In the figure join A and C; bisect AC at e, join B and e and bisect Be at h. It may be proved mathematically that a parabola may be passed through h and tangent to AB and BC at A and C, and also that at *any* point, as at n, the distance to the tangent (sn) is proportional to the *square* of the distance from A. Expressing this statement algebraically, we have

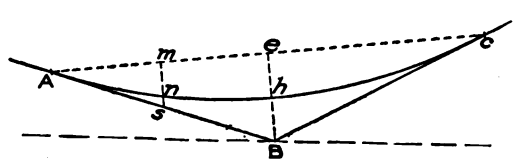


Fig. 29.

$$sn = eh\frac{(Am)^2}{(Ae)^2} \qquad (36)$$

But since, in any given case eh is a constant and $(Ae)^2$ is a constant, we may say for any one curve that the correction from the straight grade line to the curve equals a *constant* times the square of the distance from one end of the curve.

47. Numerical Example. Assume that the intersection of the grades B comes at Sta. 15 + 40; that the grade AB is − 0.6 per cent and the grade of BC is + 0.8 per cent. Then if we adopt the rule of 100 feet for each tenth per cent of change, the curve must be 1,400 feet long, must begin at Sta. 8 + 40 and extend to Sta. 22 + 40. Assume that the elevation of B is

152.50; then the elevation of A is $152.50 + (7 \times 0.6) = 156.7$. Similarly the elevation of C is computed as 158.1. Then the elevation of e is the mean of these two, or 157.4. Be is therefore $= 4.9$ and $eh = 2.45$. Then $eh \div (Ae)^2 = 2.45 \div 490000 = .000005$, the constant which is to be multiplied by the square of the distance from A to obtain the correction from the straight grade up to the curve. The elevations of the several stations is most readily calculated in a tabular form such as is given below.

A. STATION

8 + 40, elev.	$= 152.50 + (7 \times 0.6)$	$= 156.70$
9	$= 156.70 - (0.6 \times 0.6) + .000005 \times 60^2$	$= 156.36$
10	$= 156.70 - (1.6 \times 0.6) + .000005 \times 160^2$	$= 155.87$
11	$= 156.70 - (2.6 \times 0.6) + .000005 \times 260^2$	$= 155.48$
12	$= 156.70 - (3.6 \times 0.6) + .000005 \times 360^2$	$= 155.19$
13	$= 156.70 - (4.6 \times 0.6) + .000005 \times 460^2$	$= 155.00$
14	$= 156.70 - (5.6 \times 0.6) + .000005 \times 560^2$	$= 154.91$
15	$= 156.70 - (6.6 \times 0.6) + .000005 \times 660^2$	$= 154\ 92$

B. STATION

15 + 40, elev.	$= 152.50 + 2.45$	$= 154.95$
16	$= 158.10 - (6.4 \times 0.8) + .000005 \times 640^2$	$= 155.03$
17	$= 158.10 - (5.4 \times 0.8) + .000005 \times 540^2$	$= 155.24$
18	$= 158.10 - (4.4 \times 0.8) + .000005 \times 440^2$	$= 155.55$
19	$= 158.10 - (3.4 \times 0.8) + .000005 \times 340^2$	$= 155.96$
20	$= 158.10 - (2.4 \times 0.8) + .000005 \times 240^2$	$= 156\ 47$
21	$= 158.10 - (1.4 \times 0.8) + .000005 \times 140^2$	$= 157.08$
22	$= 158.10 - (0.4 \times 0.8) + .000005 \times 40^2$	$= 157.89$

C. STATION

22 + 40, elev.	$= 152.50 + (7 \times 0.8)$	$= 158.10$

In special cases it may be more convenient to note that at one-quarter of the distance from A to B the correction is $\frac{1}{16}$ of eh, at one-half the distance it is $\frac{1}{4}$ of eh and at three-fourths of the distance it is $\frac{9}{16}$ of eh.

CONSTRUCTION—EARTHWORK.

48. Slopes and Cross-sections. The construction of a roadbed of sufficient width which is level or nearly so, implies in general cuts and fills of various depths. An essential feature of the

work is that the slopes shall be such that the banks shall not give way and disintegrate, filling up the the cuts and narrowing the tops of the fills. The rate of slope is always indicated by a ratio of which the first number indicates the horizontal distance and the second the vertical. Frequently the second number is made uniformly *one* and then the first may be a fraction or a compound number. The following slope ratios will be so indicated. The rate of slope is variable, depending on the kind of soil. When a rock cut is very hard and firm, a vertical slope may be used, although on account of seams in the rock which make slipping possible after the rock has begun to disintegrate, the rock is generally taken out until the slope will average more nearly one-fourth horizontal to one vertical. As the character of the material changes from rock to earth, the slope ratio for cut must be flattened until for good, firm, loamy soil a slope of 1:1 is proper. When first excavated earth will stand at a much steeper slope than this, but the

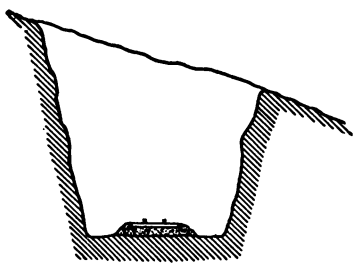


Fig. 30.

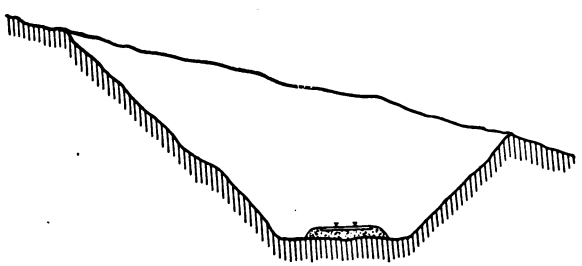


Fig. 31.

first hard rainstorm will start the disintegration which will proceed until the slope becomes about 1:1. And as it is generally cheaper to make the required excavation during the original construction, the proper slopes should be made then. Some kinds of earthy soil, such as quicksand, require even flatter slopes. Cases have been known where a cut has not ceased to slip until it had assumed a slope of about 4:1. The proper slope for a fill composed of loose

rock, as it is blasted from an excavation, is about 1:1, but when the side hill is so steep that the slope would be very long, a much steeper slope may be tolerated if some care is taken to form the stones of the fill into a rough dry wall. A fill of earth usually requires a slope of 1.5 : 1. If the material of which the fill is made is exceptionally soft, such as would require a very flat slope in cut, then a correspondingly flatter slope should be made with the fills, but in such a case it is quite possible that it would be preferable to "waste" such treacherous soil and make the fill of more suitable soil, even if it had to be "borrowed." The following illustrations will show some typical cross-sections in various kinds of soil.

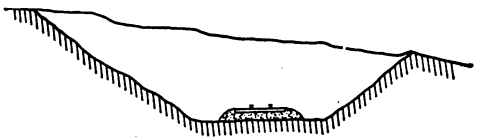


Fig. 32.

49. Width of Roadbed. A mistaken effort at economy will often be the excuse for cutting down the width of roadbed to such an amount that there is no room for adequate ditches in cuts, and the deterioration of the track due to lack of drainage is a very serious quantity. Even fills are sometimes made so narrow that the inevitable washing due to occasional heavy rains will endanger the stability of the track. A study of the standard roadbed cross-sections adopted by the principal railroads of the country shows that the average width for an earthwork cut for single track is about 25 feet. This includes about four or five feet on each side for a ditch. For double track this width is increased by about 13 feet, the usual width between track centers for two tracks. The average width for the top of a single track fill is a little over 17 feet. Sixteen feet is about the minimum for good construction.

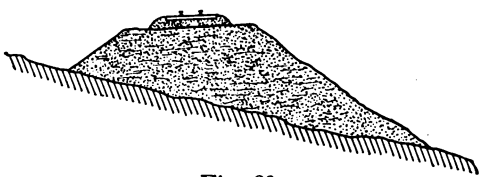


Fig. 33.

50. Constructive Details—Ditches. A well-known railroad engineer used to say that ditches were more important than ballast. A lack of good ditching will ruin a roadbed in spite of the best

ballast, while a comparatively small expenditure in ditching will largely compensate on a cheap road for the lack of good ballast. The bottom of the ditch should be from one to two feet below the bottom of the ties. The slope of the sides should not be steeper than 1:1 unless in solid rock. The bottom should be one to two

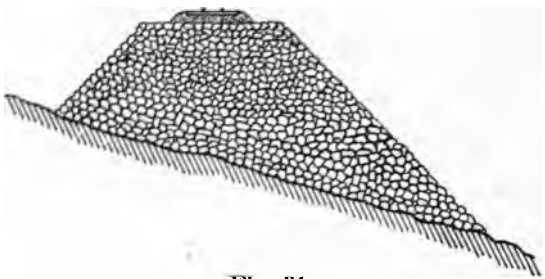


Fig. 34.

feet wide. Sometimes they are made V-shaped, but that shape is hydraulically bad and the bottom quickly fills up. *Sub-grade.* The upper surface of the earthwork, commonly called sub-grade, is usually made a level plane, but it is preferable to make two sloping planes having a crown at the center of about six inches. Rolling the sub-grade with a road roller before placing the ballast has been specified by the N. Y. Central R.R. When this is done, the water that runs through the ballast will more readily run off to the side ditches instead of soaking into the sub-soil. If the vegetable mould, which is usually found on the surface and which is the first of the excavation for cuts, is laid on one side instead of being placed at the bottom of the nearest fill and is afterward spread on the faces of the slopes of the cuts and fills, a growth of vegetation will quickly start up which will save the slopes from the effects of rain washing and will quickly repay the slight additional expense. Even an immediate sodding of the slopes during construction, although it will add considerably to the original cost, will usually save much more

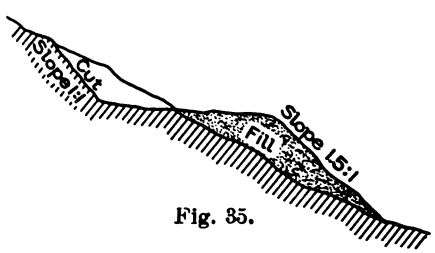


Fig. 35.

than its cost by a reduction of maintenance charges during the first few years.

EARTHWORK—SURVEYS.

51. Nature of the Problem. The mass of earthwork which is removed has an exceedingly irregular form. The surveys have the double object of staking out the limits of the earth to be removed and placed elsewhere and also of computing the volume of that earthwork. The computation of any volume means the computation of some geometrical form or combination of forms which are assumed to represent with sufficient accuracy the actual volume under consideration. If an approximate result is sufficiently close, some simple geometrical form, easily measured and easily computed, will be selected as representing the volume. But as greater accuracy is required, the more complicated must be the form. Some of the faces of the volume are simple and determinate. Sections are usually taken 100 feet apart and perhaps closer if the ground is very irregular. Such sections are plane surfaces. The side slopes are also plane surfaces. But the side representing the surface of the earth is actually rough and irregular; usually it is too irregular to be considered a plane even approximately. The surface line of each end section is considered as a broken line of one or more parts, and it is generally assumed that plane or warped surfaces connecting corresponding lines in the end sections will lie sufficiently close to the actual surface so that the error involved will be within the desired limits. It may thus be seen that the accuracy of the computation depends not only on the accuracy of the mere numerical work but even more on the judgment used by the surveyor in so selecting the places for the cross-sections and the points of any cross-section that the geometrical form assumed to represent the volume shall agree with the actual volume of earth as closely as desired. The survey therefore consists first in determining at each section the points where the side slopes intersect the surface and then the elevation and distance from the center of points so chosen that straight lines joining these points will lie very nearly in the surface of the ground.

52. Position of Slope Stakes. The slope stakes are set where the side slopes intersect the surface. As seen by the fig-

ure, these intersections depend on the elevation of the roadbed, which in turn depends on the depth of cut or fill. In Fig. 36 it may readily be seen that

$$x_l = \frac{1}{2}b + s(d + y_l)$$
$$x_r = \frac{1}{2}b + s(d - y_r) \qquad (37)$$

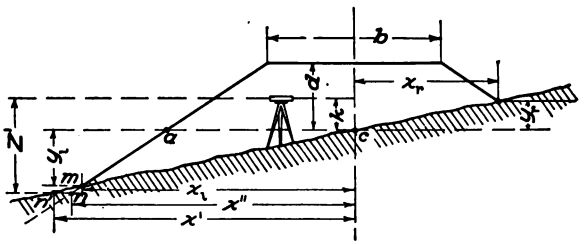


Fig. 36.

in which s is the slope ratio of the side slopes, horizontal to vertical. See §48.

Similarly it is seen in Fig. 37 that

$$x_l = \frac{1}{2}b + s(d - y_l)$$
$$x_r = \frac{1}{2}b + s(d + y_r) \qquad (38)$$

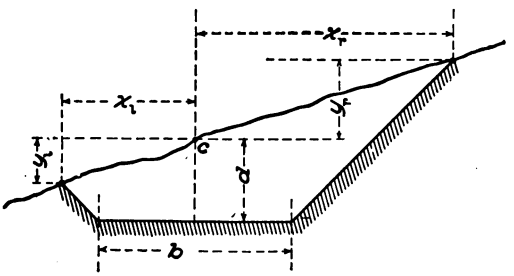


Fig. 37.

But in each of these equations, both x and y are unknown quantities, and it is impracticable to depend on a strictly mathematical solution. The simplest method is to find by trial the location of points which will satisfy the equations. An experienced man will

sometimes determine such a point in a single trial and generally the second trial will be sufficient. As a first approximation, we may note that a is at such a position that its x is $ac = \frac{1}{2}b + sd$. The added distance out to m equals the added drop times s. Assume that in Fig. 36, $d = 7.7$, $b = 20$ and $s = 1.5 : 1$. The distance $ac = 10 + (1.5 \times 7.7) = 21.55$. But experience will suggest that the required point m is *about* 8 feet lower down and therefore about (1.5×8) or 12 feet further out. As a first trial with the rod, the rod is placed at n', 34 feet out (21.55 + about 12), where a rod reading of z' (= 10.6) is found. Subtracting k (= 3.5) we have 7.1, the "y" of that point. Substituting this value of y in the first part of equation 37, we compute x_1 to be 32.2. This is the point n in Fig. 36, at a distance x'' (which is less than x') from the center. This shows that even 32.2 is too far out. Another trial is made at 30.2 feet, where the rod reading is found to be 9.3, which means that the y is $9.3 - 3.5 = 5.8$, which substituted in the equation gives $x = 30.25$. This checks so closely with 30.2, that it may be considered satisfactory. On rough ground it is an utterly useless refinement to attempt to do work closer than the nearest tenth of a foot, for the almost unavoidable inaccuracies will often have a greater effect than a change of even a tenth in such work. The above explanation is given in detail so as to show the method. Some such method is necessary for the inexperienced man, but even a short experience will enable a man to estimate the correction to his first trial very quickly and surely so that the second trial will be satisfactory, and without a detailed solution as above of all the work.

In Figs. 36 and 37, the ground has been shown as having a practically uniform slope. The determination of the slope stake is not affected essentially by the nature of the ground between the center and the slope stakes. In Fig. 42 is shown a more complicated cross-section in which the elevation of each intermediate point above the roadbed and its distance from the center must be measured. These are determined by setting up the level so that it is higher than any point in the cross-section and noting its height above the stake at the center. This rod reading added to the given center cut d gives the height of the instrument above

the roadbed. This is called the H.I. Subtracting the rod reading for any point from the H.I. gives the height of that point above the roadbed. In the case of a fill, which may be illustrated by turning Fig. 42 upside down, the level may be either above or below the roadbed. This modifies the above rule somewhat, but the same principle applies—determine the difference of elevation of each point of the surface of the ground and the roadbed.

COMPUTING THE VOLUME.

53. Common Methods. Sometimes an approximate computation of the volume of the earthwork is made from the work of the preliminary survey, so as to get an approximate idea of the amount of earthwork on a route, and therefore its cost. To do this, a more or less approximate measure of each cross-section is taken and then the distance between any two cross-sections is multiplied by the half-sum of the two areas. The sum of all such products gives the total volume. Such a method is mathematically inaccurate, but the approximation in the cross-sectional areas, and some other reasons, will probably introduce still further inaccuracies. These various methods will be described in the order of increasing accuracy.

54. Level Sections. From Fig. 38 may readily be derived the equation

$$\text{Area} = (a + d)^2 s - \frac{ab}{2} \qquad (39)$$

If A_0 is the area of the initial section and A_1, A_2, $\cdots$ A_n be the areas of the succeeding and final sections, which are at a uniform distance apart of l, then the total volume will be

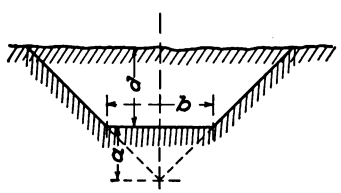


Fig. 38.

$$\text{Volume} = \frac{1}{2} l \left[A_0 + 2 (A_1 + A_2 + \cdots + A_{n-1}) + A_n\right] \qquad (40)$$

Of course l is usually 100. The $\frac{ab}{2}$ for each section is a constant, and therefore the subtractive term is simply this constant multi-

plied by the number of times the areas are used in the summation.

Example. Given the center heights set down in the second column of the tabular form. Width of roadbed, 20 feet; slope, 1.5 : 1. Then $d = 6.7$. The remainder of the solution is evident.

Sta.	Center Height.	(a + d)	(a + d)²	(a + d)² s	
42	1.4	8.1	65.61	98.41	98.41
43	2.6	9.3	86.49	129.73	259.46
44	4.3	11.0	121.00	181.50 ×	2 = 363.00
45	8.9	15.6	243.36	365.04	730.08
46	3.1	9.8	96.04	144.06	144.06

$$\frac{ab}{2} = \frac{6.7 \times 20}{2} = 67 \qquad 8 \times 67 = \begin{array}{r} 1595.01 \\ 536. \\ \hline 1059.01 \end{array}$$

$$\frac{1059 \times 100}{2 \times 27} = 1961 \text{ cubic yards.}$$

The above method invariably gives results which are somewhat too high, for the volume between two "level sections" is less than the length times the mean of the areas. When the areas are equal, the error is zero, but it increases as the *square* of the difference of the center cuts, or fills. But since sections are almost never truly level, the assumption that they are level will usually introduce an error largely in excess of the theoretical error. Sometimes the above method is used, aided perhaps by tables, by taking center cuts, or fills, from a profile and assuming that the actual volume will be the equivalent of the volume computed as above. Such a method has its value as a means of comparing two routes, but the error is apt to be very great.

55. Equivalent Sections. The following method is sometimes used when the cross-sections are irregular and especially when there is disinclination or inability to use a more accurate method. Each cross-section is plotted on cross-section paper. Then a thread (mn) is so laid that (by estimation) it equalizes the spaces above and below it (see Fig. 39). The distances out from the center of the intersections of this mean line with the side slopes are scaled from the drawing and are here called x_l and x_r. Since s is the slope ratio, $s = x_l \div mo = x_r \div np$. Then the required

area equals the area *mnop* minus the triangles *mso*, *nps*, and the "grade triangle," which means that

$$\text{Area} = \frac{1}{2}\left(\frac{x_1+x_r}{s}\right)(x_1+x_r) - \frac{x_1}{s}\cdot\frac{x_1}{2} - \frac{x_r}{s}\cdot\frac{x_r}{2} + \frac{ab}{2}$$

$$= \frac{x_1 x_r}{s} - \frac{ab}{2} \qquad (41)$$

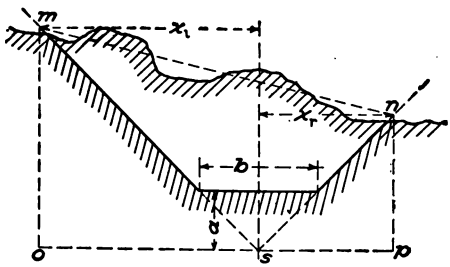


Fig. 39.

Note the simplicity of the form. When $s = 1:1$, the area equals the mere product of these two side distances minus the constant correction $\frac{1}{2}ab$. The areas being computed, the volumes are obtained exactly as in equation 40. As in the previous section, it may readily be shown that the method of averaging end areas does not give correct results except in the special case when the distances to the right (or to the left) at adjacent stations are equal, and when these distances are nearly equal the error is small. As an approximate method, it is very rapid and good. As before, the correction is usually *negative*, *i.e.*, the computed volume is too large.

56. Volume of a Prismoid. Fig. 40 represents in a perspective view a prismoid, formed between two triangles which lie in parallel planes. The surfaces which join the corresponding sides of the triangles are, in general, warped surfaces. It may be proved that the volume of such a prismoid equals one-sixth of the perpendicular distance between the parallel planes times the sum of the two end triangles and four times the middle triangle (cut by a plane parallel to the end planes and midway between them). This may be stated algebraically as follows:

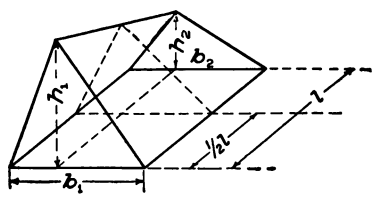


Fig. 40.

$$\text{Volume} = \frac{\text{length}}{6}\left(A_1 + 4A_m + A_2\right) \qquad (42)$$

It may also be proved that the formula holds good regardless of the values, relative or absolute, of b_1, b_2, h_1 or h_2. Therefore it holds good when $h_2 = 0$, and the prismoid becomes a wedge. It also holds good when both h_2 and b_2 become zero and the prismoid becomes a pyramid. But since the formula holds good for all these forms individually it holds good for them collectively, and since any prismoid, having bases of straight lines lying in parallel planes and with plane or warped surfaces connecting those ends, can be considered as made up of a collection of triangular prismoids, pyramids and wedges, the formula evidently holds for such a prismoid.

If, in equation 42, A_m were the mean of A_1 and A_2, then we could obtain the true volume by averaging end areas. Some of the exceptional cases where this is true have already been mentioned. In general it is a complicated and impracticable problem to compute the area of the middle section. But it is quite possible to compute the correction which must be applied to the result found by averaging end areas, and these methods will be used in the following more accurate solutions. Applying equation 42 to Fig. 40, we have

$$\text{Volume} = \frac{l}{6}\left[\frac{1}{2}\, b_1\, h_1 + 4\left(\frac{1}{2}\,\frac{b_1 + b_2}{2}\,\frac{h_1 + h_2}{2}\right) + \frac{1}{2}\, b_2\, h_2\right]$$

But the approximate volume, computed by averaging end areas, is

$$\text{Appr. vol.} = \frac{l}{2}\left(\frac{1}{2}\, b_1\, h_1 + \frac{1}{2}\, b_2\, h_2\right)$$

Subtracting the approximate volume from the true volume, we obtain the

$$\text{Correction} = \frac{l}{12}\left[(b_1 - b_2)(h_2 - h_1)\right] \qquad (43)$$

57. Three-Level Sections. When the ground is fairly uniform so that it may be said without great inaccuracy that it slopes uniformly from the center to each slope stake, then the volume may be computed from the positions of these three points and the sections are called "three-level sections." The area of such a

section $= \frac{1}{2}(a + d)\, w - \frac{1}{2}ab$. If we consider two such adjacent sections and compute the volume by the method of averaging end areas, we will obtain as the volume

$$\text{Vol.} = \frac{l}{4}\Big[(a + d')\, w' - ab + (a + d'')\, w'' - ab\Big]$$

Dividing by 27 to reduce immediately to cubic yards, we have when $l = 100$,

$$\text{Vol.} = \frac{25}{27}(a + d')\, w' - \frac{25}{27}ab + \frac{25}{27}(a + d'')\, w'' - \frac{25}{27}ab \quad \textbf{(44)}$$

When it is desired to make the computation still more accurate, the prismoidal correction may be computed as follows. We may compute separately the prismoidal correction for each of the two triangular prismoids. These two prismoids together include the triangular "grade prismoid" under the roadbed, but since there is no prismoidal correction to the grade prismoid, such correction as may be computed applies solely to the volume actually excavated. Applying equation 43 to the dimensions in Fig. 41, we have for the left side

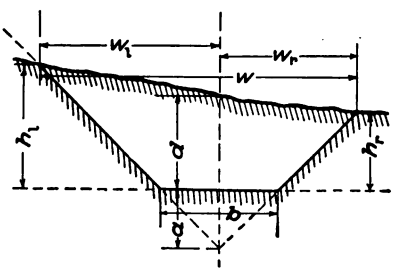


Fig. 41.

$$\text{Pris. Corr.} = \frac{l}{12}\Big[(a + d') - (a + d'')\Big]\,(w_1'' - w_1'), \text{ which}$$

$$\text{reduces to} = \frac{l}{12}(d' - d'')\,(w_1'' - w_1')$$

For the right side, we may compute similarly

$$\text{Pris. Corr.} = \frac{l}{12}(d' - d'')\,(w_r'' - w_r')$$

For the two triangles we have

$$\text{Pris. Corr.} = \frac{l}{12}(d' - d'')\Big[(w_1'' + w_r'') - (w_1' + w_r')\Big]$$

$$= \frac{l}{12}(d' - d'')\,(w'' - w')$$

Making $l = 100$ and dividing by 27 to reduce to cubic yards we have

$$\text{Pris. Corr.} = \frac{25}{81}(d' - d'')\ (w'' - w') \qquad (45)$$

An inspection of equation 45 will show that if either the center cuts or the total widths at two adjacent sections are equal or nearly so, the prismoidal correction is zero or is so small that it may be neglected. This frequently enables one to decide that the prismoidal correction will evidently be so small that it will be useless to compute its exact value. It usually happens that when $d' > d''$, w' is also greater than w''. This means that the correction computed from equation 45 will usually be *negative*, which means that for three-level sections the results computed by averaging end areas will usually be too large.

A very great economy of time and accuracy result from tabulating all the computations in earthwork. Such work can be readily reviewed to check it or to discover a supposed error. An illustration of such a solution is given below.

58. Numerical Example.

Sta.	Center.	Left.	Right.	a + d	w	Yards.		d' - d''	w'' - w'	Pris. corr.
52	3.1 C	9.6 C / 26.4	1.2 C / 13.8	11.1	40.2	413				
53	6.7 C	11.4 C / 29.1	4.2 C / 18.3	14.7	47.4	645	702	-3.6	+ 7.2	- 8
54	10.6 C	15.6 C / 35.4	7.8 C / 23.7	18.6	59.1	1018	1307	-3.9	+11.7	-14
+65	15.5 C	19.0 C / 40.5	10.6 C / 27.9	23.5	68.4	1488	1397	-4.9	+ 9.3	- 9
55	8.7 C	12.4 C / 30.6	5.8 C / 20.7	16.7	51.3	793	674	+6.8	-17.1	-13

Roadbed 24′ wide in cut. Approx. vol. = 4080 −44

Slope 1.5 : 1. Pris. corr. = −44

$a = \frac{b}{2s} = \frac{24}{3} = 8.0$ True vol. = 4041

$\frac{25}{27}ab = 178$

In the above form, $\frac{9.6C}{26.4}$ in the third column means that the slope stake on the *left* side of Sta. 52 is 9.6 feet above the elevation of

the roadbed which is here in *cut* C; also that it is 26.4 feet out from the center. This notation is also used to indicate the position of "intermediate points," the numerator of the fraction giving the depth of cut or fill (C or F) and the denominator the distance from the center. The other points in the third and fourth columns are to be interpreted similarly. Column 5 is found by adding a (=8.0) to column 2; w in each case is the sum of the two denominators in the same horizontal line; 413 (in column 6) $= \frac{25}{27} \times 11.1 \times 40.2$. A short method of performing this multiplication will be given later. The solution of equation 44 applied to this case is:

$$\text{Vol.} = 413 - 178 + 645 - 178 = 702.$$

Similarly $1397 = \frac{65}{100}(1018 - 178 + 1488 - 178)$.

and $674 = \frac{35}{100}(1488 - 178 + 793 - 178)$.

$-3.6 = 3.1 - 6.7$ and $+7.2 = 47.4 - 40.2$. $-8 = \frac{25}{81}(-3.6) \times (+7.2)$; see equation 45. Note that in this case the prismodial correction is about 1 per cent of the total volume. The errors due to inaccurate cross-sectioning will frequently be more than this. The volume 4036 cubic yards is the *precise* volume (barring the neglect of the fraction of a yard) of the prismoids given by the notes. Whether these prismoids actually represent the true volume of the earthwork depends entirely on the cross-sectioning and is entirely out of the hands of the computer.

59. Computation of Products. The products $\frac{25}{27}ab$ may be written $\frac{ab}{1.08}$. These products are always the combination of two variable terms and a constant. It thus becomes possible to construct tables which will give these products for any given height and width. Crandall's Earthwork Tables are computed on this basis. But these products are also obtained with great ease by means of a slide rule, provided it is large enough to give the required accuracy. The 108 mark, being so constantly in use should have a special mark so that it may be found without effort.

As a numerical illustration, take the first of the above cases. Set the 108 mark of the upper scale on the 111 mark on the lower scale. Then look for the 402 mark on the upper scale and note that it is nearly over the 413 mark on the lower scale. While it is possible to devise set rules to determine the position of the decimal point, it is considered that a hasty mental solution of the problem will decide the point quicker and with less chance of error. For example—the product of the two variable quantities is always divided by 1.08, which means that the final result will be a little less than the simple product of the two variables, $11 \times 40 = 440$. Therefore 413 is evidently the correct result, rather than 41.3 or 4130. The products $\frac{25}{81} xy$ are similarly obtained since $\frac{25}{81} = \frac{1}{3.24}$, and the 324 mark can be used instead of the 108. For example, the slide rule shows that $\frac{(-3.6) \times (+7.2)}{3.24} = -8$ to the nearest cubic yard. As to the decimal point—$3.6 \div 3.24$ is something more than one; therefore, the result is something more than 7.2. Therefore it is 8, rather than 80 or 0.8. If the student has neither tables nor slide rule, the multiplication of the two variables (in columns 5 and 6) and the division of the products by the constant 1.08, may be made so mechanical and systematic that it may be done quickly and accurately although it is much slower than the slide rule method.

60. Irregular Sections. The distance from the center and the height above or below the roadbed must be obtained for each break in the surface between the slope stakes. Then, in Fig. 42, by dropping perpendiculars from each point to the roadbed line, the total area is divided into a number of trapezoids, the sum of the areas of which (less the areas of the two triangles under the side slopes) equals the total area of the section. For Fig. 42, the area would be stated algebraically as follows:

$$\text{Area} = \frac{1}{2}(r+s)(f-g) + \frac{1}{2}(s+t)(g-h) + \frac{1}{2}(t+d)h + \frac{1}{2}(d+v)j + \frac{1}{2}(v+w)(k-j) - \frac{1}{2}w(k-\frac{1}{2}b) - \frac{1}{2}r(f-\frac{1}{2}b).$$

Expanding this and collecting terms, of which many will cancel out, we have

$$\text{Area} = \frac{1}{2}\Big[fs + g(t-r) + h(d-s) + kv + j(d-w) + \frac{1}{2}b(r+w)\Big]. \qquad (46)$$

Although the above equation looks as if it applied only to the particular case given, yet a little study of it will show that the terms follow a law so general that the *reduced* equation for the area of *any* section, no matter how complicated or how many points it may have, may be written out by a literal obedience of the following rule:

Area equals one-half the sum of products obtained as follows:

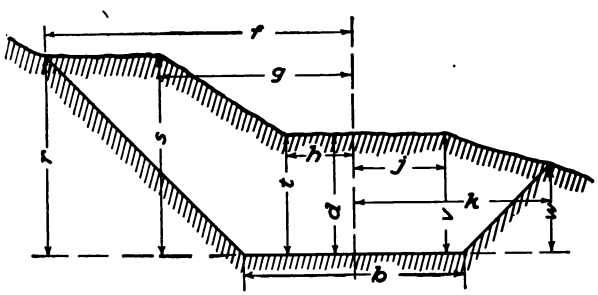


Fig. 42.

The distance to each slope stake times the height above grade of the point next inside the slope stake;

The distance to each intermediate point in turn times the height of the point just inside minus the height of the point just outside;

Finally, one-half the width of the roadbed times the sum of the slope stake heights.

The above rule may be followed literally whether there are forty intermediate points or one, or even *none*. When there are no intermediate points the terms for that side reduce to one—the distance to the slope stake times the height of the point next inside (which in this case is the center). For three-level ground (see Fig. 41), we would have three terms;

Area $= w_l\, d + w_r\, d + \frac{1}{2}\, b\, (h_l + h_r)$, which reduces to two terms

Area $= wd + \frac{1}{2}\, b\, (h_l + h_r)$.

The method of § 57 has the advantage that one of the two terms for each section is constant for all sections—in the numerical case of § 58 it is 178 cubic yards. By the above method the two terms for each section are variable. Nevertheless, when the cross sections usually have one or more intermediate points and therefore the method of § 60 must be used, and an occasional section is found with no intermediate points, it is better for the sake of uniformity to apply the above method rigidly and thereby avoid the possible confusion and error that would result by the use of another method. Probably no time would be saved by the change of method.

61. Prismoidal Correction. The above method of irregular sections is capable of being followed to its logical conclusion, and a computation for volume made which is mathematically correct, provided that it is noted on the ground how the points of adjacent sections are joined, so that the warped surfaces thereby developed shall lie as closely as possible in the actual surface of the ground. But the very fact that the cross-section is irregular implies that any longitudinal section will be correspondingly irregular, and this leads to the suspicion that a refinement in the computations may be overshadowed by a much larger but unknown difference between the volume of the assumed geometrical solid and the actual volume of the earthwork. In order to obtain a correction which is easily computed and which gives a result which is probably much nearer the truth than no correction at all, the following method is much used: Consider that *for the purpose of the correction* the ground is "three-level ground" and use equation 45. Numerical computations of volumes by both methods have shown that the *difference* is small, and is perhaps smaller than the probable error on the entire work. This method will be used in the following numerical solution:

62. Numerical Example. Volume of Earthwork in Irregular Ground. The first tabular form gives a desirable form of notes for recording the field work. Note that the station numbers

Sta.	Center } Cut or Fill.	Left.			Right.	
47	1.2 c	4.2 c / 16.4				0.8 c / 11.2
46	4.3 c	5.1 c / 17.6	6.2 c / 8.5		2.1 c / 3.4	1.6 c / 12.4
+ 42	13.6 c	20.2 c / 40.3	15.7 c / 32.4	14.4 c / 16.8	12.5 c / 10.2	9.6 c / 24.4
45	9.2 c	12.3 c / 28.5	12.7 c / 16.0	6.8 c / 6.4	9.2 c / 8.5	7.8 c / 21.7
44	3.2 c	6.0 c / 19.0	3.5 c / 8.3			1.8 c / 12.7

Roadbed 20 feet wide in cut. Slope 1.5 : 1.

FORM FOR REDUCING THE ABOVE FIELD NOTES.

Sta.	Width.	Height.	Yards.		Center height.	Total width.	$d' - d''$	$w'' - w'$	Approx. pris. corr.
44	19.0	3.5	62		3.2	31.7			
	8.3	- 2.8	- 22						
	12.7	3.2	38						
	10.	7.8	72						
45	28.5	12.7	335		9.2	50.2	- 6.0	+18.5	- 34
	16.0	- 5.5	- 81						
	6.4	- 3.5	- 21						
	21.7	9.2	185						
	8.5	1.4	11						
	10.	20.1	186	765					
+ 42	40.3	15.7	585		13.6	64.7	- 4.4	+14.5	- 20
	32.4	- 5.8	- 174						
	16.8	- 2.1	- 33						
	24.4	12.5	282						
	10.2	4.0	38						
	10.	29.8	276	667					
46	17.6	6.2	101		4.3	30.0	+9.3	- 34.7	- 100
	8.5	- 0.8	- 6						
	12.4	2.1	24						
	3.4	2.7	8						
	10.	6.7	62	675					
47	16.4	1.2	18		1.2	27.6	+3.1	- 2.4	- 2
	11.2	1.2	12						
	10.	5.0	46	265					

Approx. volume = 2372 — 156

" pr. corr. = -156

Corrected volume = 2216 cubic yards.

run up the page. By this method the "fractions" which show the height and distance out of each point are recorded in their approximate relative position when the note book is held looking ahead along the line.

It should be noted in this case that the prismoidal correction is a large percentage of the total volume. In the case of a pyramid, the correction is one-third of the nominal volume, which means that it is 50 per cent of the true volume. The foregoing numerical case gives the notes for an embankment crossing a gully, and in such cases especially the prismoidal correction is always large and should not be neglected.

63. Side-hill Work. A road frequently runs along the slope of a hill so that it is necessary to have both cut and fill in the same section. The profile at such a place will indicate little or no earthwork, but if the natural slope is steep and the roadbed wide the amount of earthwork may be considerable. Although it is possible to apply the general rule of § 60 to such cases, it is usually simpler to compute the area in each case independent of the rule, especially when the section forms a mere triangle. The areas of cut and fill must be calculated independently. When a section of cut or fill is found at one station and is not found at the next, accuracy requires a knowledge of the place where the cut or fill "runs out". That small volume must then be considered a pyramid with a given base and height. In general the end of every cut or fill implies the existence, at least for a short distance, of side-hill work. Although the volumes are usually small, yet since they are frequently of pyramidal form, the prismoidal correction is usually a large percentage of the volume and hence should not be neglected. The rule of § 61 can generally be applied.

64. Borrow Pits. This name is applied to places from which earth is taken to make an embankment when there is insufficient excavated material in the neighborhood or when the material is unsuitable for embankments. Such volumes must be measured up and paid for the same as other excavation. Sometimes the form of the excavation is literally that of a rectangular pit, in which case the simple product of the three dimensions

gives the volume. But usually it is required to slope the sides; sometimes the material is obtained by widening a cut, as in Fig. 44. If the borrow pit is very large, several cross-sections should be taken at sufficiently close intervals. If the prismoids into which the total volume is divided have substantially equal bases, the prismoidal correction will be small and may be neglected, but if the forms are pyramidal the correction should be computed It may become necessary to consider the total volume as made up of triangular prismoids and compute the prismoidal correction for each one separately.

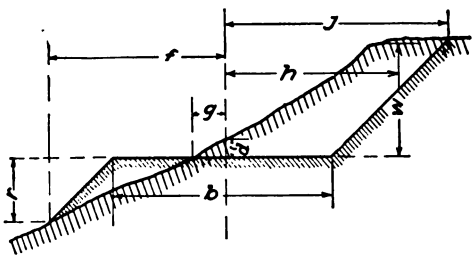


Fig. 43.

65. Correction for Curvature. The volume of a solid, generated by revolving a plane area about an axis lying in the plane but outside of the area, equals the product of the given area times the length of the path of the center of gravity of the area. When the centers of gravity of the cross-sections lie in the center of the road, where the length of the road is measured, no correction is necessary. If all the cross-sections were the same and therefore had the same eccentricity, the total volume could he computed by the above rule. But in general both the areas and the eccentricities vary from point to point, and then a theoretically exact solution would be impracticable for ordinary work if not impossible. But a sufficiently practical rule can be developed as follows: Assume a curved prismoid, of uniform cross-sections and therefore of uniform eccentricity. Call that eccentricity *e*. Let R be the radius of the center line of the road. Then the length of the path of the center of gravity is to the length measured on the center line as R $\pm$ *e* : R. Therefore we have

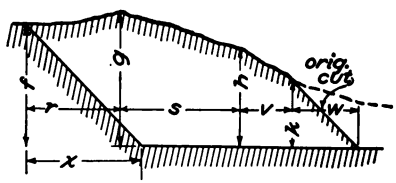


Fig. 44.

True volume : *nominal volume* : : R $\pm$ *e* : R

Therefore the true volume of the prismoid $= lA\frac{R \pm e}{R}$. This shows that the effect of curvature is the same as increasing or diminishing the area by an amount which depends on the area and the eccentricity and that the increased or diminished area may be found by multiplying the actual area by the ratio $\frac{R \pm e}{R}$. This being independent of the value of l, it is true for infinitesimal lengths. If the eccentricity is assumed to vary uniformly between two sections, the *equivalent* area of a cross-section located midway between the two end areas would be $A_m\frac{R \pm \frac{1}{2}(e' + e'')}{R}$. Therefore the volume of a solid which, when straight, would be $\frac{1}{6}l$ $(A' + 4A_m + A'')$ would then become

True volume =

$$\frac{l}{6R}\left[A'(R \pm e') + 4A_m\left[R \pm \frac{1}{2}(e' + e'')\right] + A''(R + e'')\right]$$

Subtracting the nominal volume, which is the true volume when the prismoid is straight, we have the correction for curvature as follows

$$\text{Correction} = \pm\frac{l}{6R}\left[(A' + 2A_m)\,e' + (2A_m + A'')\,e''\right]$$

As in the case of the prismoidal formula, the use of this equation implies a knowledge of the middle area. This correction is always a small proportion of the total volume and is frequently insignificant. Therefore no appreciable error is involved in making the equation read

$$\text{Curv. corr.} = \frac{l}{2R}(A'e' + A''e'') \qquad (47)$$

66. Eccentricity of the Center of Gravity. *The value of "e".* The determination of the center of gravity of a complicated irregular cross-section would be a long and tedious problem and is practically unnecessary. For the purpose of this correction it is sufficiently accurate to consider all sections as three-level sections, except in side-hill work, where they should usually be considered as triangles. In Fig. 45, the eccentricity of the center of gravity

of the whole section, including the grade triangle, may be computed as follows:

$$e = \frac{\frac{(a+d)\,x_l}{2}\frac{x_l}{3} - \frac{(a+d)\,x_r}{2}\frac{x_r}{3}}{\frac{(a+d)\,x_l}{2} + \frac{(a+d)\,x_r}{2}} = \frac{1}{3}\frac{(x_l^2 - x_r^2)}{(x_l + x_r)} = \frac{1}{3}(x_l - x_r) \qquad (48)$$

Substituting this value of e in equation 47, we have

$$\text{Curv. corr.} = \frac{l}{6R}\left[A'(x_l' - x_r') + A''(x_l'' - x_r'')\right]$$

But the approximate volume of a prismoid may be written

$$\text{Vol.} = \frac{l}{2}(A' + A'') = \frac{l}{2}A' + \frac{l}{2}A'' = V' + V''$$

in which V′ and V″ represent the number of cubic yards corresponding to the area at each station. Substituting these values in the above equation, we have

$$\text{Curv. corr. in cu. yd.} = \frac{1}{3R}\left[V'(x_l' - x_r') + V''(x_l'' - x_r'')\right] \qquad (49)$$

The value of e, found in equation 48, is the eccentricity of the whole area, including the grade triangle under the roadbed. The eccentricity of the true area is greater than this and equals

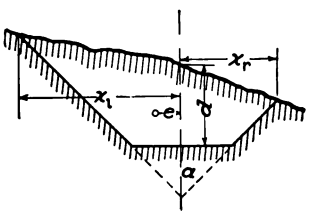


Fig. 45.

$$e' = e \times \frac{\text{true area} + \frac{1}{2}ab}{\text{true area}}$$

The required quantity (the A′ e' of equation 47) equals *true area* × e_1 which equals (*true area* + $\frac{1}{2}$ ab) × e. The value of e (given in equation 48) is a remarkably simple term, while the value of e_1 is very complicated and difficult to compute. But since we may obtain the true corrective value by using e and at the same time adding the yardage corresponding to the grade triangle to the yardage corresponding to each area, it is best to do it in this way.

For any one curve the corrective terms are all divided by the quantity 3R. If tables are not at hand, it is amply accurate to

compute that $R = \frac{5730}{D}$ in which D is the degree of the curve. Even equation 49 may be simplified somewhat in actual use. The correction at each station has the form $\frac{V(x_l - x_r)}{3R}$. 3R is usually a large quantity; for a 4° curve it is 4298. $(x_l - x_r)$ is relatively very small. Their ratio times V is never a very large quantity, and it is frequently less than unity, when, of course, it should be ignored. An approximate solution will generally show that the product $V(x_l - x_r)$ is roughly twice or three times 3R, or is perhaps less than one-half 3R, and then the corrective term for that station may be written 3, 2, or 0 cubic yards, the fraction being ignored. It is only when the curvature is excessively sharp and the eccentricity very great that the curvature correction becomes a large percentage of the volume.

The algebraic sign of the correction is most surely and easily noted from a consideration of the direction of the curvature and on which side the earthwork predominates. If the center of gravity is evidently toward the *inside* of the curve the true volume is evidently *less* than the nominal volume and the correction should be *negative*. When the curve turns to the *right*, use the form $(x_l - x_r)$; when it turns to the *left*, use the form $(x_r - x_l)$. The algebraic sign of the correction will then be strictly in accordance with its true value.

67. Numerical Example. Assume that the earthwork computed in § 58 is located on a 10° curve to the *left*. How much will be the curvature correction? Copying from the solution in § 58 the necessary data we have at once the first four columns of the tabular form. Usually the last three columns will be merely added to the form given in § 58. Since the curve is to the left, we use the form $(x_r - x_l)$. $3R = 3 \times 573 = 1719$.

Station.	x_l	x_r	Yards.	$x_r - x_l$	$\frac{V(x_r - x_l)}{3R}$	Curv. corr.
52	26.4	13.8	413	- 12.6	- 3	
53	29.1	18.3	645	- 10.8	- 4	- 7
54	35.4	23.7	1018	- 11.7	- 7	- 11
+ 65	40.5	27.9	1488	- 12.6	- 19	- 17
55	30.6	20.7	793	- 9.9	- 5	- 8

Total curv. corr. = - 43 yards.

The net volume of the mass under consideration is thus reduced to 3,998 cubic yards. A 10° curve is rather unusual. Since the correction varies directly as the degree of the curve, if the above curve were a 4° curve the correction would be only $0.4 \times 43 = 17$ cubic yards.

68. Eccentricity of the Center of Gravity of a Side-hill Section. It will generally be sufficiently accurate to consider, for this purpose, that all side-hill sections are triangular. The center of gravity of a triangle lies on a line joining the vertex and the middle of its base, and at one-third of the length of this line from the base. The eccentricity is therefore equal the distance from the center to the middle of the base of the triangle plus one-third of the base of the triangle plus one-third of the horizontal projection of that line. Applying this rule to Fig. 46, we have

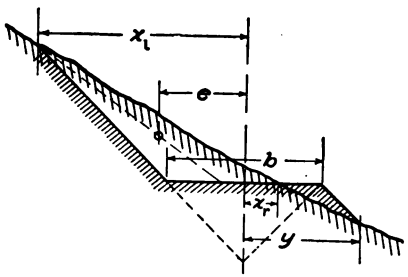


Fig. 46.

$$e = \left[\frac{b}{2} - \frac{1}{2}\left(\frac{b}{2} \times x_r\right)\right] + \frac{1}{3}\left[x_l - \left(\frac{b}{2} - \frac{1}{2}\left(\frac{b}{2} + x_r\right)\right)\right]$$

$$= \frac{b}{4} - \frac{x_r}{2} + \frac{x_l}{3} - \frac{b}{12} + \frac{x_r}{6}$$

$$= \frac{b}{6} + \frac{x_l}{3} - \frac{x_r}{3}$$

$$= \frac{1}{3}\left[\frac{b}{2} + (x_l - x_r)\right] \qquad (50)$$

It should be noted that the grade triangle is not considered in the above solution ; therefore the volume of the grade prism should not be considered in applying Eq. 49. In three-level ground the curvature correction will be zero when $x_l = x_r$, but in side-hill work the curvature correction is never zero and it may be a large proportion of the total volume.

In case the triangle lies wholly on one side of the center, it may be similarly shown that the equation may be written

$$e = \frac{1}{3}\left[\frac{b}{2} + (x_l + x_r)\right] \qquad (51)$$

This equation may be derived directly from equation 50 by considering that when both points are on the same side of the center, the algebraic sign x_r should be changed. These various cases may be generalized by saying that when the triangle lies on *both* sides of the center, e is always numerically equal to $\frac{1}{3}\left[\frac{b}{2} + (x_l \sim x_r)\right]$ in which the form should be $(x_l - x_r)$ or $(x_r - x_l)$ according to the criterion used in § 66. When the triangle is on one side only $e = \frac{1}{3}\left[\frac{b}{2} +$ the numerical *sum* of the two distances out$\right]$. Its algebraic sign should be determined as in § 66.

CONSTRUCTIVE EARTHWORK.

69. Methods of Excavating. Economical excavating depends largely on the distance to be hauled as well as on the character of the soil. Side-hill work is usually done by mere shoveling, the earth being loosened by picks or plows. When the distance that the earth must be moved becomes greater, wheelbarrows or drag scrapers become economical. Wheeled scrapers, two-wheeled carts, four-wheeled wagons, small cars drawn by horses, and heavier cars drawn in a train load by locomotives successively become more economical as the distance increases. As the magnitude of the work increases, thereby justifying an increase in the cost of the plant used, economy in the cost of loosening and loading is obtained by using a steam-shovel instead of picks and plows. When cuts are very deep, they are best excavated in "benches" whose height will depend on the method of loosening and loading.

70. Blasting. Blasting is employed not only for hard rock which can only be removed by such methods, but also as a means for rapidly loosening shale and even frozen earth. The explosives used vary in their composition from a high grade detonating explosive, such as No. 1 dynamite which consists of 75 per cent nitro-glycerine, to black powder which is, comparatively, "slow burning." Between these extremes there are a great multitude of explosive compounds, which consist of varying proportions of

explosives of the two types. It has been demonstrated that a slow burning explosive is made to "detonate" if it is exploded by means of a sufficient volume of a detonating explosive, which means that a mixture of the two kinds has a greater explosive force than the sum of the constituents exploded separately. The choice of explosive depends on the character of the rock. A hard brittle rock requires a detonating explosive which shall shatter it. A soft and tough rock is best loosened by a powder which acts more slowly. If dynamite is exploded in a soft clay rock the hole will be blown out, but the great mass of the rock is not disturbed. When the center of the mass of powder is 4 feet from the nearest surface, about 2 pounds of black powder (or about $\frac{1}{3}$ of a pound of dynamite) should be used. The amount should vary as the *cube* of the "line of least resistance," *i.e.*, if the center of the blast is 10 feet from the nearest surface the amount would be determined by the proportion $x : 2 :: 10^3 : 4^3$, or $x = 31$ pounds. In this case, since dynamite is about six times as powerful as black powder, a little over 5 pounds of No. 1 dynamite would do as well. The "line of least resistance" may not be the "nearest line to the surface" on account of seams in the rock which modify its resisting power, but the above rule is about as near a fixed rule as can be stated. For open work, especially when time is not a very important matter, it is cheaper to use black powder, but in tunnel headings where the progress of the work is limited by the progress of the drillers it is economical to use dynamite although it is more expensive.

Drilling. Hand drilling when the holes are vertical is best accomplished with "churn drills," which are heavy bars of iron shod with a steel drill, which are raised and dropped, the impact doing the cutting, the drill being slightly turned after each stroke. From five to fifteen feet of holes, depending on the character of the rock, is considered a fair day's work of ten hours. Oblique or horizontal holes must be drilled with light drills of the "one-man" type or the "two-man" type. The one-man drill is worked entirely by one man with a light one-hand hammer. With the other method, one man holds the drill, which is perhaps a little heavier and which is struck by another man, or perhaps by two, using a heavy hammer. It has been found that the light-hammer method is more economical for soft rocks, the heavy-hammer

method more economical for hard rocks, but that the light-hammer method is always quicker and is to be preferred when limited space and time are matters of importance. Machine drilling is a specialty which can only have a very brief general discussion here. Where the magnitude of the work will justify it, it is always more economical per foot of hole drilled. The plant is expensive both in first cost and maintenance; part of the expense is nearly constant regardless of the number of holes bored, and so it is only when the work is extensive that the method is advantageous, but under favorable circumstances the economy is very marked. For open-pit work individual drills are used, but for tunnel headings several drills are mounted on a "carriage" from which, after it is set, several holes may be drilled simultaneously. Compressed air is used to run the drills in tunnel work. This serves the additional purpose of furnishing a supply of pure cold air at the place where it is most needed.

Tamping. It has been found that air spaces around the explosive cause a very material reduction in the force of the explosion, therefore it is necessary to ram the explosive into a solid mass and then pack the top of the hole. Iron bars should never be used for tamping. Copper bars are sometimes used for ramming powder, but dynamite is most safely rammed with wooden bars. Clay is the best tamping material where it is available, but sand or finely powdered rock will serve very well. It has been found that when blasting under water the weight of the superincumbent water is sufficient tamping.

Exploding. On small-scale work, the blasts are generally exploded by means of a powder fuse, which is essentially a cord which forms the matrix for a train of powder, the cord being further protected by a wrapping of some sort. The better plan, and the one which is used almost exclusively for extensive work, is to explode a large number of holes simultaneously by means of electricity. A "cap" containing a small charge of fulminate of mercury, an expensive but very powerful explosive, is set in the midst of the larger mass of explosive. An electric current from a field battery is sent through each cap. As the current passes through each cap it heats a small platinum wire to redness or else causes a spark to jump across a short gap in the wire. In either

case the fulminate is exploded, which in turn explodes the main charge.

Cost. The cost of blasting is so exceedingly variable, depending on the nature of the rock, depth of the cutting, and especially on the magnitude of the work and the methods employed, that only the most approximate estimates can be here given. Under the most favorable circumstances, deep cutting, machine methods, and a rock which is brittle but not too tough, the cost may fall as low as 25 cents per cubic yard, while with hand drilling, hard and tough rock, in a shallow cut, the cost might easily rise to $1 per cubic yard. It would indicate exceptionally unfavorable circumstances, bad management, or possibly an excessive price for common labor, if the cost should rise above this figure.

71. Formation of Embankments. Experience has shown that when earth is excavated and piled in embankments its volume will at first be more than the original measured volume but that it will finally shrink to about 90 per cent of its original volume. The percentage of shrinkage is a very uncertain quantity, as it depends on the kind of earth, on the method employed in forming the embankment and on the time elapsed between construction and the measurement of what is supposed to be the settled volume. Material dumped from a trestle will first have a volume considerably in excess of its final volume and it will take several months and even years to shrink to its final volume. On the other hand, if an embankment is formed in very thin layers, each of which is packed down by the process of unloading the succeeding layer, there will be but little shrinkage after the embankment is finished, but more material than the volume as measured in the cut will be required to form that volume of embankment. Broken rock, formed into an embankment, will have a volume about 80 per cent greater than the mass of solid rock from which it is taken.

It is frequently specified that embankments are to be made to a somewhat higher elevation than the plan of the road calls for, so that the expected shrinkage will reduce the embankment to the desired level. Since the contractor is paid by the cubic yards of material *excavated* and is required to dispose of excavated material as required, it is generally specified that the amount of this

excess of height of embankments is left to the discretion of the engineer who decides the question during the progress of the work and after he has had an opportunity to judge of the material after excavation is under way. When embankments are placed on side-hills, the surface should be first plowed or have trenches dug along the slope so that the embankment shall not slip down the hill. A ditch dug at the base of each slope will drain the sub-soil and may prevent a dangerous and costly disintegration of the embankment. Thickening the layers of an embankment on the outside somewhat so that the layers will be concave upward may also present sliding of the layers on each other. When the plans call for a very long and high embankment, it is sometimes best to construct first a trestle and operate the road over it. The trestle should have a life of at least five or six years, and during that time material can be brought from some excavation, perhaps several miles away, where it was perhaps loaded with a steam shovel, hauled by the train load, dumped with an "unloader," and allowed all the required time to settle, the whole being done for a cost per yard far less than it would have cost during the original construction. The method has the added advantage of permitting the road to be quickly opened for traffic and permitting it to quickly get on an earning basis, for such a trestle can be built more quickly than a very high embankment.

72. Classification of Earthwork. One of the most fruitful sources of legal contention between a contractor and his employer is the classification of excavated material when the work is paid for according to the classification of the material excavated. It is not only true that there is an insensible gradation from the softest of earth to the hardest of rock, but a material which is very hard when first exposed will sometimes crumble up after a very short exposure to the atmosphere. It is even true that some kinds of rock which are very soft when first taken out harden after exposure to the air, but this class of phenomena never has any influence on mere blasting for excavation. To avoid these disputes, some railroads require their contractors to satisfy themselves as to the character of the material to be excavated and then to make a single bid per yard which shall include whatever material is encountered. With all its advantages, this throws all the uncertainty on the con-

tractor, and unless the competition is very great and the bidding close the contractor will usually add so much to cover that uncertainty that the railroad will pay more than it would on a classified basis.

The classification is usually made threefold—(1) solid rock, (2) loose rock, including shale and hard-pan, and (3) earth. Solid rock includes only such material as cannot be removed except by blasting, when it is found in masses exceeding one cubic yard. Loose rock includes boulders which are more than one cubic foot in volume and less than one cubic yard; also stratified rock occurring in layers of not more than six inches, when they are separated by strata of clay; also all material (not classified as earth) which *can* be loosened with a pick and bar and which "*can* be quarried without blasting although blasting may occasionally be resorted to." "Earth" includes all material not considered above—boulders not over one cubic foot in volume, all clay, sand, gravel, loam, decomposed rock and slate, and all materials which can be loosened for loading by a plow with two horses, or such as one picker can keep one shoveler busy. A brief consideration of the above classification, which is compiled from the best authorities available, shows the infinite opportunities for dispute as to classification.

TUNNELS — SURVEYING.

73. Character of Surveying. There are few kinds of surveying for engineering work where accuracy is of such high financial value and where it is so difficult to accomplish as it is in tunnel work. By the very nature of the case a tunnel is usually located in a region where it is very rough and all the surface surveys must be made on very steep slopes where accurate measurements are exceedingly difficult. The surveys in the tunnel itself are made in cramped quarters where light is artificial and the atmosphere is perhaps smoky. The difficulties will be elaborated as the methods for obviating them are discussed. Tunnels are generally excavated from each end. A very small error at either end will accumulate, especially if the tunnel is very long, until when the two headings meet there may be an offset which might actually necessitate a small reversed curve in the alignment. Therefore only the most refined measurements for distance, the

most refined leveling between the ends of the tunneling and the repeated measurements of all horizontal angles or the most precise prolongation of lines are to be used. All such work should be repeated and checked until the probable error of the work is so small that such error as may remain has no financial importance. The cost of such refined work is amply justified, because the lack of it may result in an error whose financial value might be very great.

74. **Surface Surveys.** The relative position of the two ends of the tunnel is first determined, *i.e.*, the azimuth and length of a line joining the two ends and the relative elevation. Usually a line is run on the surface which will be at every point exactly over the center line of the tunnel. When the tunnel is perfectly straight throughout, this is comparatively easy. Any curvature in the tunnel complicates the surveying greatly. A permanent station, from which a long sight can be run into the tunnel, is placed at each end. Then intermediate permanent stations are set so that adjacent stations are intervisible. These stations are first set approximately on line and then by repeated adjustments they are all set exactly on line. Any intermediate shaft can then be located from the adjacent stations.

Distance. The distance is sometimes determined, as in geodetic surveying, by triangulation and the measurement of a base line. Some of the great Alpine tunnels have been measured in this way. But for simpler work a direct measurement is made with a tape. Since the slopes are usually very steep, it becomes impracticable to hold any very great length of the tape truly horizontal. It is then also necessary to plumb down from the down-hill end of the tape to the ground. This is troublesome and also introduces an element of inaccuracy. And therefore "slope measurements" are often made, measuring the slope distance between carefully marked points and at the same time determining the difference of elevation. A simple geometrical calculation then determines the true horizontal distance. These marks may consist of needles set in wooden plugs supported on ordinary surveying tripods.

Levels. The above method includes the leveling. But if the ordinary method of leveling is used, especial care must be taken, since the slopes are very steep and the vertical distances to be overcome very great.

75. Underground Surveys. Station marks, corresponding to the stakes of ordinary surveys, cannot usually be placed in the bottom of the tunnel since they would be very quickly disturbed or covered over with debris. If the tunnel is timbered, the easiest method is to place the marks on the timbering, but this should not be done unless the timbering is very firmly in place and is not liable to be shifted. The better plan is to drill a hole in the roof of the tunnel, insert a wooden plug, and then set in the wood a small hook or nail which marks the exact point. Occasionally such marks are placed on the side of the tunnel. When placed in the roof there is the advantage that a plumb line, which must be illuminated by a lantern, may be swung from the hook or nail. A still better device is a plumb bob hung by a pair of cords attached to a "gimbel joint" on the bob. The bob has a little reservoir for oil and a wick exactly in the center which will furnish a flame which may be seen as far as necessary and which may be bisected by the cross hairs of the transit with great accuracy. Such "sights" can be reproduced whenever desired with great confidence that there is no appreciable variation. When a mere plumb line is used to sight at, it must be illuminated by some sort of lantern. This may be done by using a lantern with a ground glass which is placed behind the line, which is seen by its contrast against the illuminated background. If an ordinary lantern is used, it should be placed nearly in front of the line and pointing away from the transit, so that, without being seen from the transit, it illuminates the face of the line which then shows light against the darkness of the tunnel.

The leveling must, of course, be done with the level rod inverted so as to obtain the distance from the station point *down* to the line of sight. Of course this makes a corresponding difference in the calculations which must be carefully watched to avoid a blunder due to this change. This may be avoided by always placing a minus sign before any rod readings so taken, and then following the old rule of *algebraically* adding backsights and subtracting foresights and intermediate sights. Various devices are required to meet special conditions which test the inventive ingenuity of the engineer.

76. Surveying Down Shafts. In the case of very long tunnels it sometimes seems advisable to sink shafts at one or more points on the line of the tunnel, and when they have been sunk to the required depth, proceed to dig out the tunnel in each direction. For such work it becomes necessary to determine, at the bottom of the shaft, elevation, distance and alignment. If the shaft is vertical, as is usually but not always the case, the elevations are most readily carried down the shaft by means of a steel tape by methods which are obvious. Distance, which means in this case the longitudinal position in the alignment of the road of any given point, is readily transferred from the surface to the level of the tunnel by a very obvious application of the results of the next process to be described. Transferring the alignment with accuracy requires the utmost care and ingenuity. In principle it is very simple. Two heavy plumb bobs are hung on steel wires which are long enough to reach from the surface to the tunnel. At the surface they are placed on a line. Theoretically they should be on a line at the level of the tunnel. If a transit is so placed in the tunnel that its line of collimation passes simultaneously through both wires, it is in the line of the tunnel. Such is the simple outline; some of the difficulties are as follows:

Although the wires are set as far apart as possible along the line of the tunnel, the distance is absolutely limited by the size of the shaft. Any minute error in the location of these lines (say eight feet apart) will be greatly magnified when the headings are run out 6,000 or 7,000 feet in each direction from the base of the shaft, as was done in the case of the Hoosac tunnel. The currents of air up a tunnel shaft have considerable effect in swaying the wire from a true vertical. In the case of the Tamarack shaft, 4,250 feet deep, the wires were 0.11 foot farther apart at the bottom than at the top. The discrepancy *in that direction* had no effect on the alignment, but if the wires had an error whose combined effect in that direction was 0.11 foot, the lateral error while unknown was perhaps as much or more. The uncertainty was therefore in that case very great. Incasing such wires for the whole distance in a box reduces the effect of air currents. The plumb bobs are swung in pails of water or oil at the bottom and their locations noted as carefully as possible, taking the mean posi-

tion of the vibrations which cannot be altogether overcome. Marks are then set at the bottom of the shaft (but in the roof of the tunnel) from which plumb lines can be hung. A transit can then be set by trial so that its line of collimation simultaneously passes through both wires.

TUNNEL DESIGN.

77. Cross-Sections. The variety in the cross-sections which have been adopted is due to the fact that there are no absolute requirements which determine the design except in a general way. If the tunnel passes through such very soft soil that there is excessive pressure the form should be circular or nearly so. While the size of the rolling stock is in one sense a limitation, yet the clearance should be considerable, partly for the reason of allowing something for a possible settlement of the lining. A majority of the sections used have a semi-circle or semi-ellipse surmounting a rectangle or trapezoid. Even when the ground is so soft that lining is required not only at the top but also at the sides and bottom, the same general shape will be used except that the straight lines will be replaced by arcs of circles concave to the center of the tunnel. Illustrations of cross-sections will be shown under a subsequent section.

A tunnel almost invariably strikes one or more veins of water which immediately begin to discharge into the tunnel, which thereafter becomes the drainage outlet for such water. This necessitates an adequate provision for drainage. In a double track tunnel the drain will usually be placed between the two tracks, but with single-track tunnels they must necessarily be placed on each side. Fig. 48 will illustrate this feature.

78. Grade. Many tunnels are situated at the summit of two grades, which are very probably the ruling grade of the road. In such a case it is possible to make the ends of the tunnel at practically the same level and have no grade in the tunnel except a slight grade for drainage. There should be no grade summit in the tunnel. Grade for drainage should never be omitted—about 0.2 per cent grade is required. But tunnels are frequently necessary as parts of a grade which is very possibly the ruling grade of the line. In such cases the grade should be very materially re-

duced while running through the tunnel. The atmospheric resistance in a tunnel is greater, the rails are apt to be wet and slippery and the tractive power therefore less, while the consumption of the limited supply of oxygen by the locomotive and the poisonous fumes cast off, especially when the engine is working hard, is a source of actual danger to the engine crew and even to the passengers. Therefore a generous reduction of grade should be made, although the precise amount of compensation required is hardly computable.

79. Lining. The lining required varies from no lining, such as may be permitted when the rock is so firm that it will be absolutely self sustaining and will not disintegrate upon exposure to the weather, and a lining of the very heaviest and strongest cut-stone masonry which should be used when the ground is subject to extensive settling. This condition is far worse than any mere fluid pressure. Many American tunnels have been constructed with a permanent lining of timber, such as is illustrated in Fig. 47. In other cases the cross-section of a tunnel has purposely been made somewhat larger than necessary, so that when the timber lining required renewal a masonry lining could be built inside of the timber lining without encroaching on the required clear cross-section and without requiring any disturbance of the timber

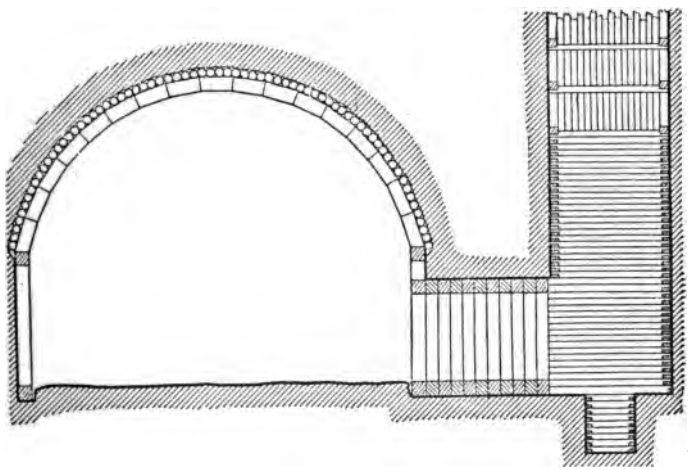


Fig. 47. Connection with Shaft, Church Hill Tunnel.

lining. In this way the heavy expense of the masonry lining could be deferred until a time when the road would probably be better able to pay for it. True economy requires the best of cement masonry. When, on account of an unintentional fall of rock outside of the nominal excavation lines a space would be left between the lining and the line of the excavation, such space should be filled with broken stone well packed in or even with concrete or solid masonry.

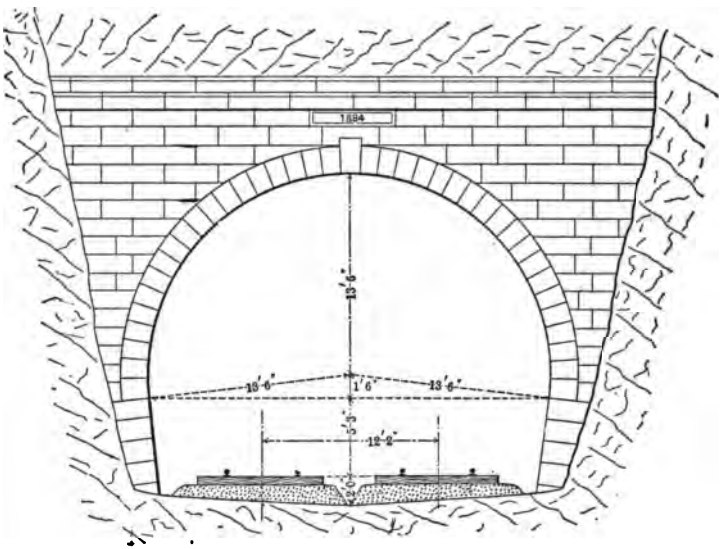


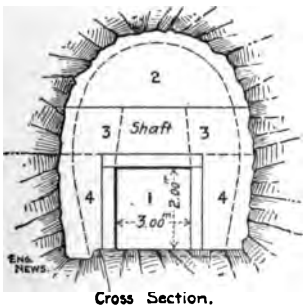
Fig. 48.

80. Portals. Although no calculations can be made to determine the forces acting on a portal, it is readily seen that they are sometimes very great, as they must often prevent a tendency of the face of the mountain to slide down over the tunnel outlet. In Fig. 48 is shown a typical portal. These are sometimes made very elaborate architecturally, but the leading feature of the design must be its massiveness. It must act as a retaining wall against the direct action of the slope. If there is also a considerable stretch of open cut at the entrance to the tunnel, then the design is really simplified by walls on the sides of the cut which will act as buttress walls to the portal. Some of the most difficult construction of a tunnel may occur at the portals. It is here that the thickness

of the natural "roof" of the tunnel runs out to practically zero. Considerable thickness is required before it will become self-sustaining enough to give opportunity to place the timbering. The surface soil may also be so loose that the excavation below starts a landslide. Therefore a very heavy timber frame must be constructed outside of the line of the proposed portal and must be very heavily braced to withstand a probable tendency to a landslide. In one case a shaft was sunk a short distance from the portal; tunnel excavation and permanent masonry lining was at once commenced, running back toward the portal. As the surface was approached the thin roof was so thoroughly supported that no serious difficulty was encountered from a landslide.

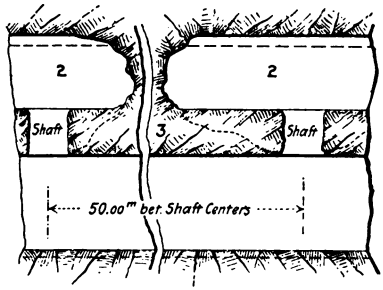
TUNNEL CONSTRUCTION.

81. General Principles. A large majority of tunnels require a lining because the material through which they are excavated cannot be depended on to be self-sustaining. Except in sub-aqueous



Cross Section.

Fig. 49.



Longitudinal Section.

Fig. 50.

work, all material is self-sustaining over a small area and for a short time, a time long enough so that after a small area has been exposed a support even though temporary may be placed which will prevent a fall at that place. Since there are all gradations in material from the hardest of rock to the softest of quicksand, there are likewise gradations in the methods to be adopted, in the promptness with which timbering must be placed to support exposed areas and also in the extent of area which may be safely exposed before timbering is placed. All methods agree in excavating one or more headings in advance of the full sectional excavation.

These headings are sometimes made at the top, sometimes at the bottom and sometimes at the two lower corners. One good effect of such headings is to drain the soil in advance of the main excavation and thus facilitate the subsequent work. These headings are then enlarged until at last the full sectional area, including that required for the lining, is obtained. The construction of the lining follows closely, so that in a stretch of perhaps less than 80 feet may be seen all stages of the work, from the initial heading to the finished tunnel completely lined.

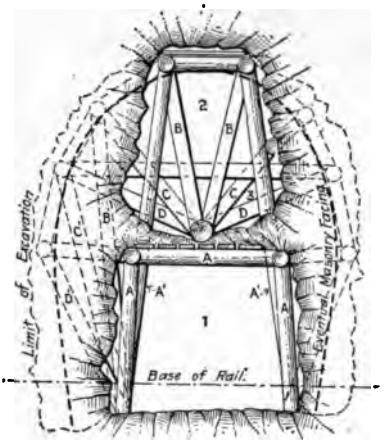


Fig. 51.

82. Methods. The limitations of this paper will not permit a complete discussion and description of the various methods which are used in this work. Some illustrations are shown to give the student such a grasp of the general principles involved that a generous application of common sense may enable him to accomplish some of the plainer and simpler problems. The timbering must be so designed and placed that there will be little or no tendency for the pieces to slip on each other and that any added pressure will only bind the framing still tighter together. The timbering should never fail except by absolute crushing, and its cross-section should be made such that it may withstand any probable pressure. An inspection of the illustrations will illustrate this.

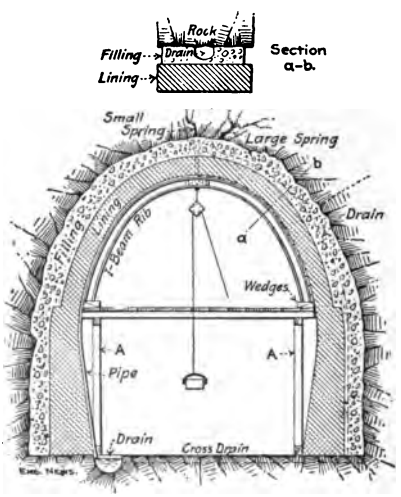


Fig. 52.

Much of the difficulty of tunnel work arises from the limited space in which the work must be done. A well-devised system of removing excavated material as rapidly as it is loosened and of handling the materials for the lining and placing them in position is therefore an absolute necessity. The use of small cars on rails is usually advisable. With a tunnel of any considerable length, artificial ventilation during construction is necessary, especially if blasting is required. As before mentioned, compressed air may be used to operate the drills for blasting and this may supply the need. But where no blasting is required, and sometimes even when compressed air is used, ventilation by fans is necessary. The fans and engines for operating them are of course placed outside the tunnel and the fresh air is discharged from a pipe where desired.

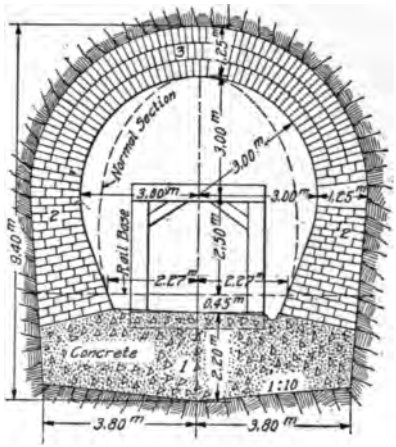


Fig. 53.

TRESTLES.

A trestle consists of two essential parts, the sub-structure framework and the floor system. Since the floor system is essentially independent of the sub-structure, it will be separately described. There are two systems of building the sub-structure, by piling and by timber frames.

83. Pile Trestles. These are limited in height to the length of a single pile which may safely be used. The length of pile required must include the necessary depth to which they must be driven. On this account 30 feet above the ground is about the limit of height of a pile trestle. With exceptionally long piles higher pile trestles might be built, but framed bents would be preferable. Usually four piles are considered sufficient for single track, although more are sometimes used. The inner piles are always made vertical but the outer piles are sometimes battered so

as to give the trestle a greater resistance against a lateral thrust. For a high trestle (greater than 10 feet) this thrust is best taken up by sway bracing. The piles are surmounted by a cap which is generally 10×12 in., or perhaps 12×12 in. A still better form of cap is the "split cap," which consists of two pieces bolted together as shown in the detail of Fig. 56. Other methods of joining the cap to the piles are illustrated herewith. The construction using drift bolts is perhaps the cheapest and most quickly erected, but it has the disadvantage that repairs are difficult, and if the trestle is merely temporary it is almost impossible to remove it without ruining the timber for future use. The mortise and tenon joint is perhaps the most common for good practice. The piles should be not less than 14 in. in diameter at the butt and 7 in. at the top, exclusive of bark, which should be removed before driving. The soft durable woods such as cedar, cypress, pine and redwood are best for piles that are not driven in a stream where they may be subject to the blows of floating ice. The oaks are stronger but are less durable in the ground. The caps are preferably made of hard wood such as oak or yellow pine. They should be about 14 ft. long for single track. The sway braces are generally 3×12 in. and are usually spiked with $\frac{3}{8}$-in. spikes 8 in. long.

84. Pile Driving. Piles are usually driven by means of a hammer weighing 2,000 to 3,000 pounds, which is raised between guides to a height of perhaps 25 feet and allowed to drop onto the head of the pile which is suitably set between the guides. A very cheap way is to raise the hammer by horse power, and then loosen a clutch which allows the hammer to fall freely. A still better way is to use a portable engine which winds the hoisting rope around a drum. Sometimes the falling hammer is required to draw the rope and unwind the drum as it falls. On the one hand, this obviates the use of a clutch and even permits more rapid blows, but on the other hand, the force of each blow is very materially weakened and the method may be used by a dishonest contractor to falsely indicate a high resisting power of the pile. Excessive driving has been known to fracture a pile underground and render it almost useless. The action of the hammer splinters the top of the pile, causing it to "broom." This action very greatly reduces the effectiveness of the driving. This is largely prevented by chamfering

off the top of the pile and driving on a wrought-iron ring, which has a section of about $\frac{1}{2}\times 2$ in. and of a suitable diameter. The frequent removal of all crushed wood from the head of the pile by means of an adze is amply justifiable in spite of the delay caused. Piles should be driven until their resistance as indicated by the penetration for a single blow is as great as is required. The most commonly used formula is that known as the "Engineering News" formula, which when used for ordinary hammer driving is as follows:

$$R=\frac{2\,wh}{s+1} \qquad (52)$$

In this formula R is the *safe* load on the pile, w is the weight of the hammer, both in pounds, h is the height of the fall in *feet*, and

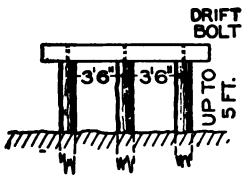


Fig. 54.

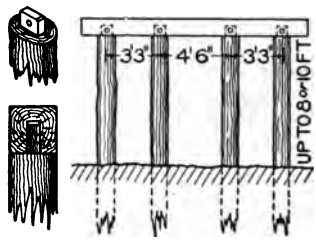


Fig. 55.

s is the penetration in inches of the pile during the last blow. Sometimes the average penetration during the last five blows will give a more reliable value.

Example 1. A pile was driven with a 2,500-lb. hammer until the total penetration during the last five blows was 13 inches. During those blows the hammer dropped 23 feet. How much is the safe load?

$$\frac{2\,wh}{s+1}=\frac{2\times 2{,}500\times 23}{(\frac{1}{5}\times 13)+1}=\frac{115{,}000}{3.6}=31{,}944 \text{ pounds.}$$

Example 2. It is required to drive piles with the above hammer until the indicated resistance is 25,000 lb. What should be the average penetration during the last five blows, the fall being then 22 feet?

$$25{,}000=\frac{2\,wh}{s+1}=\frac{2\times 2{,}500\times 22}{s+1}=\frac{110{,}000}{s+1}$$

$$s=\frac{110{,}000}{25{,}000}-1=3.4 \text{ inches.}$$

Another form of pile driver is that known as the "steam pile driver." This consists essentially of a hammer which is directly attached to a piston in a steam cylinder. The hammer, weighing about 5,500 pounds, is raised the height of the cylinder, which is about 40 inches, and then falls freely. Although the fall is so much less the blows are very rapid—about 75 to 80 per minute. The practical effect of this is that the soil does not have time to settle between the blows and the penetration is more easily accomplished, while the ultimate resistance is as great as before. On this account the constant "1" in the denominator in equation 52 is changed to 0.1 and the formula then becomes

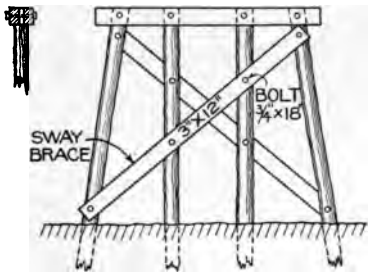


Fig. 56.

$$R = \frac{2\,wh}{s+0.1} \qquad (53)$$

FRAMED TRESTLES.

85. General Form. Although there are multitudinous variations in the details of construction, a very large proportion of framed trestles are constructed substantially in accordance with the typical design shown in Fig. 57. The outer posts are generally battered about 1:6; the cap and sill are mortised to the posts, although split sills and caps can be used advantageously. The sway bracing should be bolted on. Although the mortise and tenon joint are most commonly used, there are many other designs. The "plaster joint" is one of the most common. This consists of two pieces of 3-in. plank which are placed on each side of the joint as in Fig. 58, and are bolted through and through. This form has the merits of

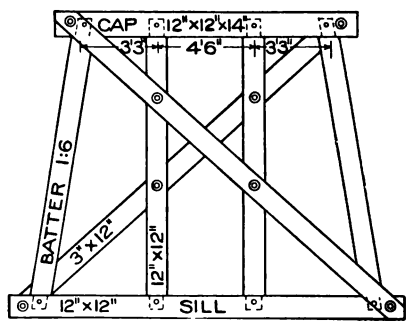


Fig. 57.

cheapness and facility for taking out and renewing any decayed or injured piece. Iron plates are sometimes used in a similar manner. Dowels and drift bolts are also used, but as mentioned before have many objections.

86. Multiple Story Construction. A single-story framed trestle should not be made over 25 or 30 feet high. Additional height is obtained by dividing the height into two or more stories. Then since all the upper stories must be of uniform height the odd amount must go to the lower story, as is illustrated in Figs. 59 and 60. Some plans have these stories absolutely independent of each other. This simplifies the construction and makes repairs easy, but the trestle will be lacking in stiffness. These illustrations should be studied with special reference to the design of the lateral bracing of the individual bents and also the longitudinal bracing of the trestle as a whole. Note that the lateral bracing always runs to some point where two or more pieces intersect, and when possible it is so designed that even the intermediate points are a common point for several pieces. A thorough bolting at these points greatly stiffens the structure. The span between the bents varies from 10 feet to 18 feet. For high trestles economy requires that the number of bents shall be reduced as much as possible, which means that the spans should be increased. But this increases the requirements for the floor system, and also the load to be carried by each bent. 18 feet is about the safe limit for railroad rolling stock on untrussed wooden floor beams.

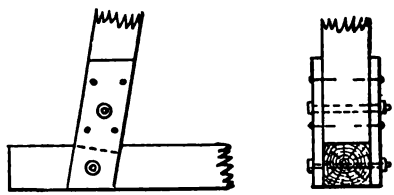


Fig. 58.

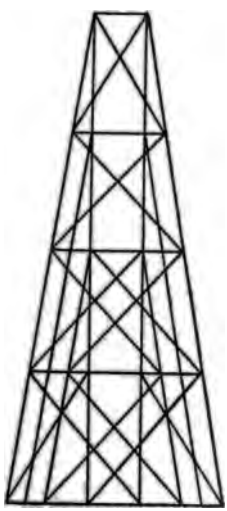


Fig. 59.

87. Foundations. Trestles are frequently to be classed as "temporary" structures. Such will justify the use of a foundation of a more temporary character than could be tolerated for per-

manent work. When time is important and the ground soft, piles are sometimes driven and sawed off a little above the ground. They are so placed that a pile comes as nearly as possible under each post of the trestle. Of course, such foundations must be considered as very temporary in character, as they will speedily decay to such an extent as to render them unsafe. Locust or chestnut are preferable for this purpose.

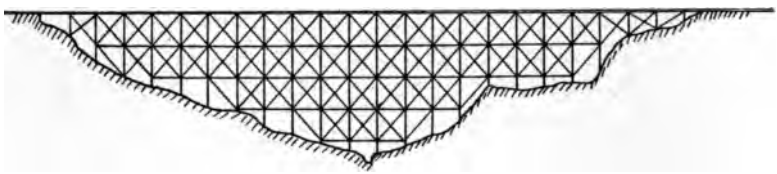


Fig. 60.

Another form which is even easier to construct, but which is, if possible, still more subject to decay, is the "mud-sill" foundation. The sill of the trestle is set on a number of timbers placed transverse to the sill, the timbers being about 12×12 in. × 6 ft. If the ground is very soft even these timbers may be set on two or more long timbers, laid parallel to the sill, as shown by the dotted lines in Fig. 62.

When the trestle is intended as a permanent structure, and especially when it is intended to ultimately replace it with a steel viaduct, a stone foundation may be used. If built of rough rubble the wall should be about 4 feet thick. If the masonry was of a better class the wall need not be so thick, but the cost would be about the same. Usually a single continuous wall would be built, but if the trestle is very high the usual batter adopted for the side posts will make the sill very long. With some designs of trestles, depending, however, on the plan of the posts, it is permissible to save some masonry by omitting portions of the wall between the center and the ends.

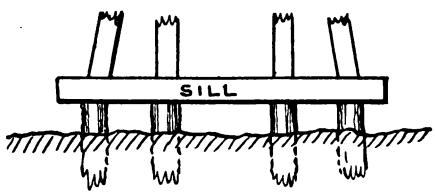


Fig. 61.

88. Abutments. At each end of a trestle the natural surface usually approaches the grade line by a slope. If stone foundations were built for the bents, then stone abutments would be built which would act as a retaining wall for the last few feet of rise and which would support the last stringers. When piles are used an abutment such as indicated in Fig. 63 is used. When no piling is used, an abutment may be made in the form of crib work. Sometimes one end of the last line of stringers is merely buried in the earth or is supported on a "mud sill." Of course, all of these latter methods should be considered as temporary. The danger in them lies in the chance of these places being neglected and the decay unnoticed until the decayed timber suddenly gives way and a costly accident is the result.

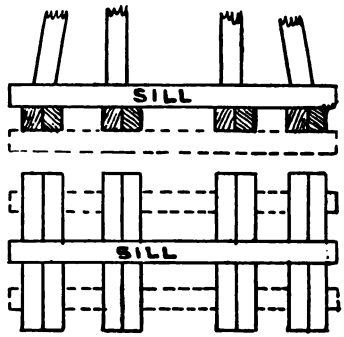


Fig. 62.

TRESTLE FLOOR SYSTEMS.

89. Stringers. The design of stringers depends somewhat on the cost and practicality of obtaining timbers of the length and thickness that theory would call for as the most economical size. Sound timber of the required length, and more than 16 or 17 inches in thickness, is scarce and correspondingly costly. The required transverse strength for stringers is, therefore, obtained by taking as large pieces as may be readily obtained, setting them on edge or with the largest cross-sectional dimension vertical, and then bolting two or more of them together side by side. Two timbers, each 8×16 in., bolted together side by side

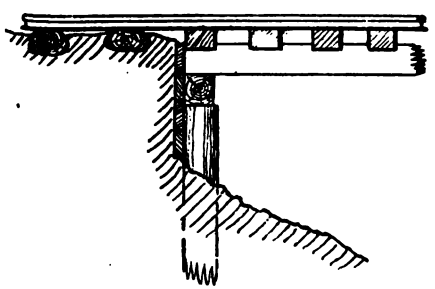


Fig. 63.

with the 16 in. dimension vertical, are practically as strong as a 16×16 in. timber, and are very much easier to obtain in a sound condition. These stringers should preferably extend over two spans, the lines "breaking joints." This requires pieces from 20 to 32 ft. in length. The pieces of each line should be separated by "separators," which are sometimes cast-iron spools, 1 or 2 in. long, which are strung on the bolts, or are sometimes made of pieces of plank about 6 feet long. Bolts are run through the stringers and separators. The plank separators thus serve to tie the consecutive stringers together. The chief object of the separators is to permit air to circulate freely around the timbers. Placing the rough sawed timbers side by side would allow water to soak in and be retained, so that decay would be very rapid. The design of stringers is susceptible of exact calculation for the transverse strength required, but as this is a direct application of the subject of "Strength of Materials," the method of design will not be elaborated here, except to call attention to the fact that the stringers must be designed to withstand not only transverse strains, but also shearing and crushing across the grain where the stringer rests on the cap. A very high and narrow stringer might have sufficient transverse strength, but might be so narrow that it would fail by shearing along the neutral axis. The same stringer might also have such a small area where it rests on the cap that the safe limit of crushing across the grain might be exceeded. The safe values to be used with various kinds of wood for these various stresses may be found in many handbooks. The following dimensions have the approval of very extensive practice:

Clear Span.	Number of pieces under each rail.	Width.	Depth.
10 feet.	2	8 inches.	16 inches.
12 "	2	10 "	16 "
14 "	3	10 "	16 "

90. Corbels. A corbel in a trestle is the name applied to a timber placed on the cap of the trestle bent and on which the stringers rest, Fig. 64. The argument in favor of their use seems to be that they greatly increase the area of pressure on the seat of the stringer. They can also be utilized to bind together two abutting

stringers. But although the crushing of the end of the stringer may be prevented, the area of contact between the corbel and the cap must be considered, to determine whether crushing might

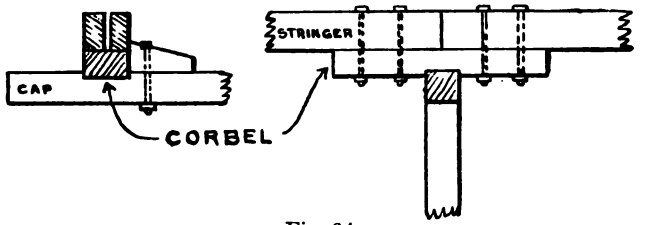


Fig. 64.

occur there. There is great diversity of opinion regarding their use. Many standard designs do not use them.

91. Guard Rails. These are timbers varying in size from 5 × 8 in. to 8 × 8 in. which are placed near the ends of the ties. They are usually notched about 1 inch at each tie so that they really form tie spacers and thus prevent the ties from becoming

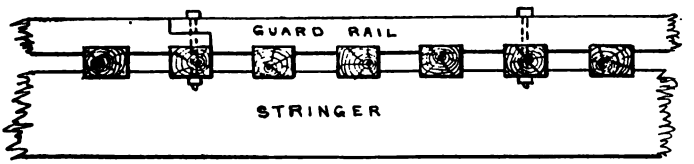


Fig. 65.

displaced if a car becomes derailed on the trestle. They should be bolted to the ties at every third or fourth tie. There are various methods of jointing the ends of abutting pieces. The method shown in Fig. 65, is perhaps as good as any.

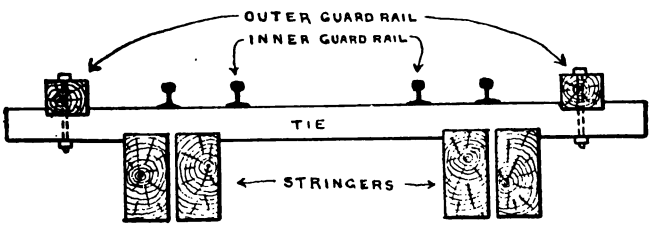


Fig. 66.

The name guard rail is also applied to the inner guards which are placed about 10 in. inside of each rail, Fig. 66. These are usually

the ordinary T-rails used for traction. These rails are relied on to keep the cars on the trestle if they should become derailed. If a car should become so displaced that the wheel reached the outer wooden guard rail, it would probably catch on it, slew around and jump over. Therefore the sole function of the outer wooden guard rail is to keep the ties spaced and in place.

92. Trestle Ties. Trestle ties are always made of sawed timber. They are longer than ordinary ties—usually 9 to 12 feet. The depth is frequently much greater, with the apparent idea that they may act as a flooring that will support the rolling stock if it should become derailed. For a similar reason the spacing is made very much closer—generally equal to or less than the width of the ties. Sometimes even the ties are notched on the underside where they rest on the stringers. Some plans have a stringer run under each guard rail. Then bolts will be run through the stringer, tie and guard rail at every third or fourth tie. If the ties have been notched down on the stringers and the guard rail is notched down on the ties, then these bolts will tie the whole system immovably together.

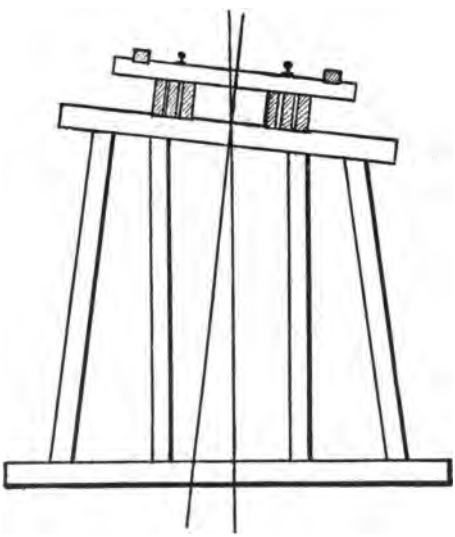


Fig. 67.

93. Super-Elevation of the Outer Rail on Curves. Locating a curve where a trestle is also necessary, is in general very objectionable, but it is sometimes unavoidable. The objection lies not only in the fact that a very considerable force is required to guide the train in its circular path, but the force is variable, depending on the variable speed, and there are apt to be oscillations of unknown force which will still further rack the trestle. Nevertheless these forces must be provided for as closely as possible. If all trains moved along the trestle at precisely the same speed,

the problem would be comparatively simple. The whole floor system could be designed to resist the thrust due to the forces developed at that speed. But since the speed may vary down to zero, and the train start from rest while on the trestle, which of itself will introduce new strains, the construction which is best for the highest speed is not the best when the train is standing on the trestle, or when it is starting, and *vice versa*. A few of the many designs which have been used will be illustrated, together with abrief comment on the advantages and disadvantages of each design. The required super-elevation of the outer rail and the method of computing it will be discussed in the chapter on track work and track laying.

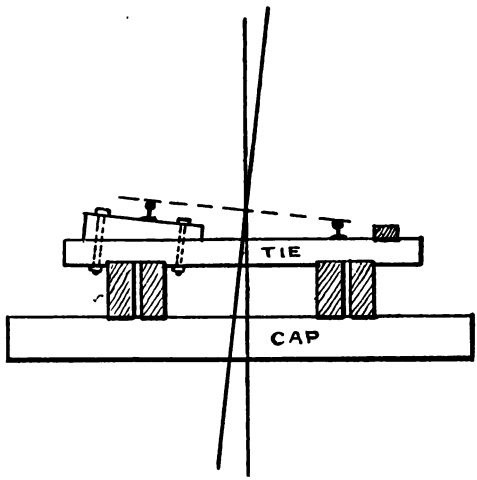


Fig. 68.

(*a*) *Inclining floor system and cap; sill horizontal; outer posts longer, Fig. 67.* The construction of the trestle bents is more complicated, but that of the floor system is simplified. Since the stringers do not stand vertically there is a tendency for them to twist when the train is stationary.

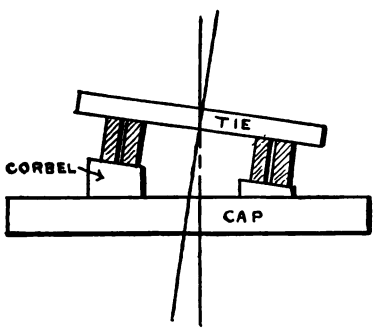


Fig. 69.

(*b*) *Placing wedges under the ties at each tie.* Two or more wedges are required for each tie. Each wedge is bolted by two bolts. The number of pieces is very great, but there is the advantage that the ties are not notched or weakened in any way. If for any reason a different super-elevation is desired the wedges are all that need be changed.

(*c*) *Placing a wedge under the outer rail at each tie, Fig. 68.* This is similar to the last method, but requires fewer pieces. If the super-elevation is slight, either very long spikes must be used or lag screws may be used which will run through the wedges into the ties. For a greater super-elevation the wedge must be fastened, as shown in the illustration.

(*d*) *Corbels of varying height, Fig. 69.* The whole floor system is tipped as in *a*, but the trestle bent is as usual, with cap and sill horizontal. In all such cases, where the axis of the post is vertical, the lateral bracing of the bent should be made extra heavy. It should be especially noted whether the center of pressure under extreme conditions reaches the sill too near the outer end of the sill.

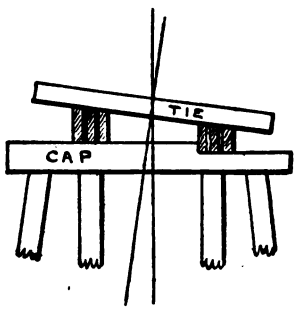


Fig. 70.

(*e*) *Tipping the whole bent on its foundation.* The advantages and disadvantages of the method under some conditions are obvious.

(*f*) *Notching the cap, Fig. 70.* The method is mentioned on account of its frequent use, but the disadvantages are very great. The cap is weakened by the notching. Either the stringer or the tie must be notched at each tie. Usually the tie would be hopelessly weakened if it were notched sufficiently, and, therefore, the stringers must be notched. The method is costly in construction and objectionable when made. The above methods are types of a great variety of plans of construction which have been suggested and tried.

94. Protection Against Fire. One of the strongest objections against the use of trestles is the danger from fire. Sparks from the locomotive or wayside fires kindled by tramps and others may destroy them, or, what is still more dangerous, may slowly eat into the timbers until they are weakened beyond the danger line, and yet, because the effect of the fire is not apparent to a careless inspection, it may result in an appalling accident. The danger from falling coals from the locomotive firebox is largely obviated by constructing a solid floor or trough on the stringers, the trough being filled with ballast and the ties set in the ballast

as usual. Another method is to cover the stringers and caps with sheet metal. A very long trestle generally deserves the protection of a special watchman or track walker. Means for fighting a fire when discovered are provided by reservoirs of water, made perhaps from halves of oil barrels, which are placed on the trestle at intervals of 300 feet. Three or four ties are made about 4 feet longer than the usual length. These form the floor of a platform, which, when provided with a railing, forms not only a place for the barrel, but also a refuge bay for the track walker, who may be on the trestle when a train is passing.

95. Choice of Timber. When a railroad is being run through a virgin country where timber is plentiful and there is frequent occasion for trestles, it pays to take a portable sawmill to the spot and saw the timber as required. Under such conditions any one of the various kinds of timber which are ever used for building purposes will answer. If necessary, the cross-sections can be increased to correspond with the reduced strength of a weaker wood. But when the wood must be transported a considerable distance and it is practicable to choose among various kinds of wood, the selection should be made according to favorable qualities. Ties and guard rails should, if possible, be of oak. Stringers should be made of oak or pine. Since one of the chief uses of corbels is to relieve a dangerous pressure across the grain they should be made of the hardest wood obtainable, such as oak, hickory or ash. The bents of a framed trestle may be made of almost anything, but oak, pine or fir are preferable when obtainable. If the sills are liable to become buried somewhat in the ground so that rain will not readily be shed, then some wood like cedar, which is very long-lived under ground, might be preferable, but the strength as posts will be somewhat less than that of oak. The chemical treatment of timber for trestles is seldom used, except for trestles which are partly immersed in water where the *teredo navalis* is found. Trestles are usually considered to be so cheap and temporary that conditions which would justify the additional expense of chemical treatment would also justify the immediate construction of a permanent structure of steel or stone.

On the folding plate, Fig. 71, is shown the standard plans for a framed trestle as adopted by the Great Northern Railroad.

Many of the details shown will verify those already mentioned, while in other cases the variations in detail represent practice equally good. The plate is well worthy of a long and close study.

CULVERTS.

96. Pipe Culverts. The scarcity of stone suitable for making a "box" or "arch" culvert has led to the adoption for many localities of pipe culverts, the pipes being made of tile or iron, Fig. 72. Pipes have several very great advantages. Their form is hydraulically better than any rectangular form and the surface is

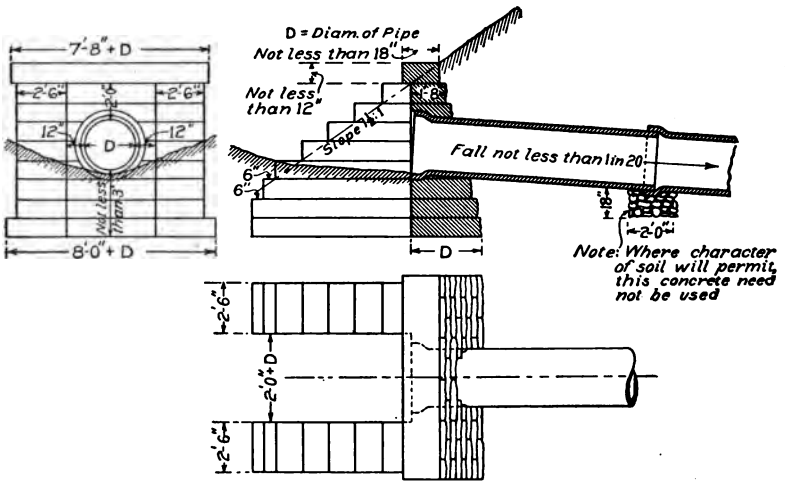


Fig. 72. Pipe Culvert.

usually very much better than an ordinary masonry culvert. Therefore they will discharge a far greater volume of water than a box culvert of equal area. They are very easily placed without skilled labor. Sometimes they are set inside of a wooden box culvert temporarily placed during the construction of the road. When one pipe of the size which it is desired to use has insufficient area two or more pipes may be used side by side. This feature is of special value when the head room between the bed of the stream and the grade line is limited. Iron pipe usually has such inherent strength that there is little need for special care in securing a foundation for the pipe. A little block of concrete at each joint is sufficient for ordinary cases, but tile pipe requires a secure foundation.

The danger to the pipe does not lie so much in the mere static pressure of the earthwork embankment above it as in the effect of settlement of a "green" embankment. If the pipe is laid on the natural soil, which might be tolerated if it is very firm, a bed should be carefully scooped out so as to fit the pipe as closely as possible. A better plan is to place a thick layer of broken stone or brickbats and ram them to the proper form as a bed for the pipe. A still better plan is to place a layer of concrete under the whole length of the pipe. The required slope of the pipe depends somewhat on the accuracy of the laying and on the permanency of the work. A slope of 1 per cent is ample, provided the grade be made and maintained uniform, but the effect of settlement may be to change such a grade to a negative grade, which would prevent the water from being carried off. Some standard plans there-

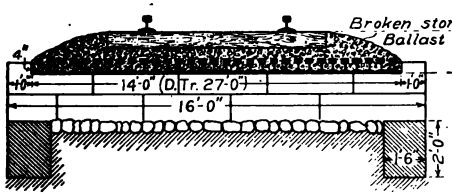


Fig. 73.

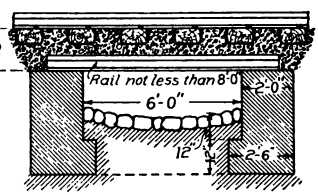


Fig. 74.

Old-Rail Culverts.

fore require a grade as steep as 1 in 20. At each end of the pipe there should be a substantial head wall of masonry. Some standard plans make this wall very large and heavy with elaborate wing walls. These are justifiable on the grounds of preventing the water at the upper end from scouring around the ends of the head wall or of preventing the outflowing water from scouring away the bed of the stream and thus undermining the lower head wall.

An iron pipe can be used if necessary very close to the ties, but a tile pipe should have a cushion of at least three feet between the tile and the ties. The joints in the pipe should always be caulked. Clay puddle is much used for this purpose and when it is of good quality and the work well done, the results are satisfactory, but if clay puddle cannot be obtained it is better to use hydraulic cement. The cost of the cement is an insignificant item considering the value of the result.

97. Old-Rail Culverts. These have an especial value when the head room between the bed of the stream and the rails is small, and when it is also necessary to provide for a considerable flow of water. The old rails, even when worn out as rails, still have a considerable strength as girders and a continuous layer of them is amply strong enough to carry the roadbed and the traffic over a six-foot opening. The rails may be bound together by means of tie rods run through the webs of the rails, but they may also be confined by stones at each end of the seat course on each abutment, Figs. 73 and 74.

Another advantage of this form of opening, over the common plan of supporting the ties on stringers or steel girders, is that in this plan the ballasted roadbed is continuous. This is a great advantage both from the standpoint of smooth riding and of safety.

98. Cattle Passes. When an embankment crosses a farm, cutting it in two, it becomes necessary for the road to provide a passage way through the embankment for the use of cattle and farm wagons. The cost of such a structure is compensated by the relief of the company from damages due to the cattle crossing the road at grade. These openings are sometimes built as large stone arch culverts or as old-rail culverts, especially if there is liable to be a storm-water flow through them. Another method is to set two trestle bents at the requisite distance apart; 3-inch planks are set behind the bents to hold the earthwork embankment; the stringers are notched down so as to take up the thrust of the embankment. This method naturally applies to embankments which are from about 8 to 15 feet in height. The disadvantage incident to all wooden structures set in earth also applies here. There is also the disadvantage of a break in the continuity of the ballasted roadbed, as well as the danger due to an accident from fire destroying or weakening the structure. When the head room is limited, a first-class permanent construction can best be obtained by the "old-rail" method or something similar.

EXAMINATION PAPER